Harmony-Seeking Computations

A Science of Non-Classical Dynamics Based on the Progressive Evolution of the Larger Whole

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All illustrations are to be found at the end of the paper, starting on page 61.

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In this paper, I am laying out a new form of computation, which focuses on the harmony reached in a system. This kind of computation in some way resembles certain recent results in chaos theory and complexity theory. However, the orientation of harmony-seeking computation is toward a kind of computation that finds harmonious configurations, and so helps to create things, above all, in real world situations: buildings, towns, agriculture, and ecology.

This way of thinking about computation, though mathematical at base, is closer to intuition and artistic feeling than the processes we typically describe as "computations". It is also more useful, potentially, in a great variety of tasks we face in building and taking care of the surface of the Earth, and quite different in character since it is value-oriented, not value-free. It has everything to do with beauty. The harmony that is sought in these computations is indeed what we otherwise call "beauty". But the result of harmony-seeking computations are not merely pretty or artistic. In most cases, they are also better functionally and technically. That is why they are important.

Examples are taken from farming, art, architecture, biology, physics, astrophysics, drawing, crystallography, meteorology, dynamics of living systems, and ecology.

PREFATORY REMARKS

Overview

The scientific importance of harmony-seeking computations arises in three ways.

First, there are a number of natural phenomena, especially those occurring in complex systems, that are approximated by present-day explanations, but which have annoying mismatches with reality. In a few cases that I describe in this paper, for example *Acetabularia* (pages 19-21), snow crystal formation (pages 44-46), cloud formations (pages 37-39), simulation of tree growth (pages 39-40), cosmology of large-scale structure in the universe (pages 25-29), and group formations in bird flight (pages 35-37), there is a substantial possibility that the discipline of harmony-seeking computation can contribute something to solving puzzles and unanswered questions where solutions have not, so far, come successfully from the fields themselves.

Second, there are cases where the conscious use of harmony-seeking computations improves on current practice of design, planning, and ecology, in fields where it is our business to make beautiful and harmonious structures, but where, too often, contemporary architects and planners using current techniques fail to do this (for example, wind turbines (negative) pages 17-19, Tokyo apartment building (positive and negative) pages 22-23, Boston housing (positive) pages 47-50, drawing by Nakano (positive) pages 23-24, tree bole seat (positive) pages 13-14).

Third, there are other cases where study of traditional building processes, and the structures they generate, give insight into the nature of the harmony-seeking computations themselves (for example, St Mark's Square (pages 32-33), hayricks (pages 16-17), the Parthenon (page 43-44)).

This paper is based on results first presented in the four-volume work *The Nature of Order*.[1] The essential results, providing the underpinning for the current paper, are the following: There is a structure, visible in any given part of the world, which we may call *the wholeness*. The wholeness is an abstract mathematical structure, existing in space. It captures what we may loosely consider as the global structural character of a given configuration, in itself and in relation to the world around it. The wholeness is a structure that exists at many levels of scale, and covers the interrelationships of the configurations at different scales. The primary entities of which the wholeness structure is built are *centers*, centers that become activated in the space as a result of the configuration as a whole. Centers typically have different levels of *strength* or coherence. The coherence of a configuration is caused by relationships among centers. In particular, there are 15 kinds of relationships among centers that increase or intensify the strength of any given center. These 15 properties are listed below, and define the way that configurations within a configuration help each other. Within this scheme, unfolding of new configurations is a natural process, and can be understood and followed. We thus have a basis for making computations about unfolding. These are somewhat similar to the bifurcations that have been observed and analyzed in complex non-linear systems,[2] but they are much richer and more complex than the theory of bifurcations can at present contemplate. Unfolding occurs as a result of wholeness-extending (W-E) transformations. These W-E-transformations are combinations and sequences of 15 possible spatial transformations based on the 15 properties that determine how coherent centers may be built from one another. An advanced computational theory of these W-E transformations does not yet exist, but it is my aim here to show how unfolding is built from these transformations, and how the outline of a new (computational) theory of unfolding can be established.

In this context the term "wholeness-extending" relates to preserving and extending the structure of *wholeness*. The term "structure-preserving" is sometimes used in mathematics to refer to transformations that preserve some *particular* structural aspect of a given system, but by convention this particular aspect may be arbitrarily chosen. In my use of the term, it means that the given transformation preserves the *whole*, and is not arbitrary, but dependent on the observer's ability to *see* the whole.

Wholeness

To make more explicit what the term "wholeness" means, I describe the phenomenon of wholeness as it appeared in a series of psychological experiments. In studies performed at the Harvard Center for Cognitive Studies some years ago, my co-workers and I discovered a curious phenomenon.[3] When confronted with simple black and white strips, and asked to arrange them according to their relative similarities, different perceivers chose one of two very different strategies. These two possible strategies for looking at similarity are illustrated below:

Figure 1 Sequential-digital: Reading the strips left to right
Figure 2 Figural-holistic: Seeing the strips as patterns

In Figure 1 we see five columns, organized and defined by a left-right reading of the patterns, somewhat akin to the Dewey decimal system of organizing books in a library or a system of ordering binary numbers according to their left right reading. The right hand end of each pattern plays virtually no part in placing it on the board, and the right hand ends are random with respect to black and white. The left hand end in each group starts with the same figure, two black squares in the first column, or one black square in the second column, and so on. The *gestalt*, or pattern as a whole, plays no part in the arrangement of the strips on the board.

In Figure 2, we see various groups, and what ties together the patterns in a group, is the *overall* kind of pattern they have. One group has steady alteration of black and white squares. Another has a single chunk of black, with chunks of white on either side. Here the basis of classification is just the opposite from what is going on in Figure 1. The patterns are grouped according to their overall configuration, *as a whole.*

Now, one may observe that these figural arrangements are more interesting. To a scientist, it is plain that this way of looking at the figures is more useful in the study of physical systems, such as the pattern of emerging galaxies, or the morphological development of embryos. As such we may consider the figural classification as deeper, or more interesting. The sequential basis for classifying patterns might interest a computer scientistconcerned with binary codes, but otherwise will be relatively less interesting, and less likely to be important.

However, our experiments revealed a remarkable fact. Our experimental subjects were young women students at Radcliffe college, and, by definition therefore, intelligent, analytical, and selected for good performance in academia. About 85% of these Radcliffe students chose the sequential kind of arrangement. Only about 15% used the figural kind of arrangement.

The form of classification used by each student was spontaneous, and not “led” in any way by the experimenters. The experimental instructions asked only that they play with the strips on the gray board supplied, and place those that look similar near each other on the board. Each student made a unique and different kind of arrangement – no two were exactly the same. But they fell into these two sharply different kinds of arrangements.

It was startling to find out that Harvard Radcliffe students with high IQ, and high academic performance, should spontaneously tend to create arrangements that were based not on the inherent figural configurations in the material they were looking at, but instead on an arbitrary classification scheme that was external to the nature of the material. This was especially startling in light of the fact that either scientific training or artistic training would tend to look towards the figural aspect of the strips as their most salient feature.

A further interesting fact emerged. We became interested in the possibility of training people to see the figural aspects of the strips. Because of our experiments we had invented, in effect, a way of calibrating a perceiver's way of looking at the strips, by objectifying their mode of perception in the arrangement that perceiver created, and using that arrangement as a definition of how that person was seeing the world.

We therefore began a second series of experiments in which we tried to give people various pre-experiment tasks, selected, we thought, in such a way that they would tend to induce "figural" perception in the subject. One, for example, was "creative play" in which subjects were encouraged to make beautiful arrangements with the strips. Others involved cognitive tasks of various kinds. After giving training to the subject, we would then ask them to make the arrangement on the gray board, always grouping patterns which "looked alike to them". We always used subjects who had never seen the material before, so there was no residue from an earlier experience.

To our surprise virtually none of these techniques (we tested more than a dozen different ones) had any appreciable effect on the way the subjects perceived the strips. In the end we discovered only one technique that had an effect. In this technique we showed each subject a single one of the strips, flashed on a screen, by itself, and allowed them to study it and absorb its character. We then put on the screen, for only half a second, an array in which the same 35 strips were tightly packed in a random fashion, and offered the subject 5¢ if they could find the one we had been showing them. After a few attempts our subjects learned to do this. It was quite impossible to do it by reading the strips one at a time. One could read no more than a couple before the array disappeared off the screen. The only way to find the targeted strip was to do the opposite: instead of an earnest focused type of perception, one had to lean back, mentally, and try to see the whole board in a single glance. This required great concentration, but was inherently unfocussed and relaxed if it was to work. When people learned this technique (which they discovered spontaneously – we never explained it to anyone) they were then able to see the targeted strip, and tell us where it was in the array, and so earned their 5¢.

The experiment gave us a strong indication that there is such a thing as "seeing the whole". And that the "whole", or "wholeness", that is seen is, plainly, an objectively present structure. It is not always easy to have the ability to see it, and students trained in analytical thinking are less likely than others to be able to see it at all: we found in other experiments that both very young children, and the mentally retarded, had much greater facility for seeing wholeness than the highly educated Radcliffe students. To teach someone to see the wholeness, one had to break down the analytical habits of a lifetime of education.[4]

The ability to see the wholeness is a prerequisite for the material contained in this paper. It is also, I believe, a prerequisite for seeing many of the key phenomena that are now an essential part of physics and biology, just as it is a prerequisite for seeing art with understanding.

Fifteen Properties of Wholeness

In *The Nature of Order*, I have once again taken up the study of wholeness that had first occupied me in these Harvard experiments 40 years earlier.[5] Among other fundamental aspects of wholeness I focus on certain structural features which appear to be underpinning the wholeness structure, as it appears in the geometry of physical things. In *The Nature of Order* I report on my observations of 15 structural features that appear again and again in coherent systems, and appear to play a major role in establishing the wholeness of these systems. The 15 are summarized very briefly here.

1. STRONG CENTERS. Wholeness is composed of centers, and centers arise from wholeness. A given wholeness is coherent to the extent that the centers in it are coherent.[6] Centers are recursive in structure. Each center that exists acts to strengthen other centers, larger and smaller.

2. LEVELS OF SCALE. When a configuration contains centers, these centers are associated with centers at a range of sizes that occur at well-marked levels of scale. The scale jumps between levels are small: in coherent systems the centers of different sizes are often in size-ratios of 2 to 1, 3 to 1 and 4 to 1. If the jumps are larger – for example 10 to 1 or 100 to 1 – without intermediate levels, the coherence tends to fall apart. This means that in coherent structures, the ladder or hierarchy of levels has evenly spaced rungs, and is continuous and smooth.

3. THICK BOUNDARIES.[7] Strong centers typically, though not always, have thick boundaries around them. These thick boundaries may exists in 1-, 2- or 3-D, and are themselves made of smaller centers that have the levels of scale relation to the larger centers being surrounded. These boundaries typically form a transition zone of interaction, allowing physical, chemical, or biological processes to occur without contaminating the centers being surrounded. The boundary is often on the order of one scale jump smaller than the thing it surrounds, thus may be equal to the diameter or half the diameter of the thing surrounded. Boundaries help form the field of force that creates and intensifies a center; they surround, enclose, separate and connect.

4. ALTERNATING REPETITION. When repetition of similar centers occurs in a coherent system, the centers typically alternate with a second system of centers, thus forming a double system of centers with a beat or rhythmic alternation, from the positive space between the repetitions. Centers intensify other centers by repeating in a rhythm; when a second system of centers also repeats, in parallel, it intensifies the first system by providing a kind of counterpoint, or opposing beat.

5. POSITIVE SPACE. In coherent systems, there is no “background”, or figure and ground. Instead, every bit of space is coherent, well shaped; and the space between coherent bits of space are also coherent and well-shaped. Thus every bit of space swells outward, is substantial in itself, and is never the leftover from an adjacent shape – like ripening corn, each kernel swelling until it meets the others, each one having its own positive shape caused by its growth as a cell from the inside. The positiveness of space is difficult to pin down exactly, but it is like a weak kind of convexity, or quasi-convexity. In systems where the space is positive, the principal elements of space are nearly all quasi-convex, and the pieces of space between these elements are also quasi-convex.

6. GOOD SHAPE. This describes a particular, coherent quality of the particular shapes that occur in or around a coherent center. This kind of “good” shape is somewhat unusual, and is marked by the fact that the shape itself is made up from multiple coherent centers which together form the shape, and of other coherent centers which together form the shape of the space around the shape.

7. LOCAL SYMMETRIES. Strong centers often have strong symmetries, and local parts of space with strong symmetries are typically strong centers.

This feature binds together smaller centers within the whole, further creating coherence.

8. DEEP INTERLOCK AND AMBIGUITY. This occurs where coherent centers are "hooked" into their surroundings, making it difficult to disentangle the center from its surroundings. Often there are ambiguous zones which belong both to the center and to its surroundings, again making it difficult to disentangle the two.

9. CONTRAST. Every center relies to some degree on the contrast of discernible opposites, and on its differentiation from the ground where it occurs. It is intensified when the ground, against which it is contrasted, is clarified and itself becomes made of centers: all this differentiation arises from the degree or sharpness of contrast that is attained. Note, though, that too-sharp a degree of contrast is offset by NOT-SEPARATENESS, below.

10. GRADIENTS. Centers are generated and strengthened by gradients of size, shape, or quality. Thus any quality among a system of centers that varies systematically produces a gradient, and this gradient, by pointing to a particular center, helps to build that center and to intensify its coherence.

11. ROUGHNESS. In coherent structures we usually find a rough arrangement and repetition of centers rather than exact repetition in shape, spacing and/or size. Thus apparently similar centers are different according to context, allowing each part to be adapted to the geometric constraints around it, thus modifying details of the repeating structure as it needs to be. Texture and imperfections are generated, and in part create the possibility of true uniqueness and life.

12. ECHOES. Within coherent configurations there are often deep underlying similarities or family resemblances among the elements. These similarities are often characterized by typical angles, and typical curves, so that they generate what appear to be deeply related structures, sometimes so deep that everything seems to be related.

13. THE VOID. In the most profound centers that have perfect wholeness, there is often at the heart of the structure a void that is large, undifferentiated, like water, infinite in depth, surrounded by and contrasted with the clutter of the structure and fabric all around it.

14. SIMPLICITY AND INNER CALM. Essential to the completion of a coherent whole is a quality of simplicity. Every structural feature that is unnecessary has been removed, so that the remaining structure has slowness, majesty, quietness. Everything superfluous has gone.

15. NON-SEPARATENESS. Connectedness; we experience a living whole as being at one with the world around it, not separate from it. This means that when not-separateness exists, visible strands of continuity of line, angle, shape, and form, connect the inside of a living center with the parts of the world beyond that center, so that it is, ultimately, impossible to draw a line separating the two.

These 15 properties are thoroughly explained in Book 1: *The Phenomenon of Life*.[8] In Book 2: *The Process of Creating Life*, these structural features are also shown to form a basis for the structure-preserving transformations that create life and coherence as configurations unfold.[9]

INTRODUCTION

The central issue of this paper is geometric adaptation. In many real world systems, both in nature, and in those places where human beings form communities with animals, plants, and other human beings, the central observable is a close-knit adaptation of the system elements, usually arising over time, and most often expressed in the intricate geometry of the system.

This close-knit geometric adaptation has not yet been a major focus of scientific study, because it eludes simple algorithmic formulations.[10] That is not because it is more complex, or too complex to be modeled. It happens, rather, because the elements of such adaptation are so extremely simple, and so rooted in common sense, that they nearly elude the algorithmic and algebraic formulations that we view, wrongly, as more sophisticated.

For example, if a farmer places a row of fence posts, then runs a top rail, braces it here and there where it seems needed, allows it to relate in natural ways to declivities in the ground, or to the present of nearby trees, this is supremely ordinary; it is characterized entirely by common sense, and often by the farmer's ability to pay attention to the situation of each post, each rock, each bit of soil, each slope – and do it right. But this oh-so-simple process eludes algorithmic formulation, because algorithmic formulation is inappropriate, not tailored to this tasks, and almost certainly unhelpful in grasping what is really going on.

That is not to say that such a sensitive fence-building process is childish or unimportant. On the contrary, the character of this minute, step-by-step adaptation is vitally important, and we have been ignoring it at our peril. But we do not presently have a model that emulates this process. As a result, the ability to perform in the real world according to such a process has been worn away and destroyed by processes that are largely bureaucratic – often too bureaucratic – but also, in their essence, algorithmic. The planners, building officials, construction companies, engineers, who have redefined everyday processes during the last 100 years, have been working in a broad context of algorithmic thinking and yes-no thinking. And they have, without explicitly intending to, destroyed a far more subtle process.

Until that subtle process is acknowledged, and redefined in modern terms, it will not have the status it requires to play an effective role in modern society. The deep adaptation that nourishes the physical world requires this kind of adaptation. And this adaptation process, if we choose to think of it as computation, is a highly sophisticated computation, yet performed on real sticks and stones, and nevertheless potentially huge in the depth and subtlety of its results.

The progress of an evolving (unfolding) natural landscape, or the development of an embryo, have similar qualities. As cell division progresses, new cells take shape within the context of the surrounding cells, and, at the same time, adapt, so that they are shaped by these surroundings, and simultaneously play an active role in shaping their surroundings. Again this process has not been modeled. One reason for that is the prevailing assumption that what is going on in the world is too often a nearly random aggregation of simple mechanical processes, with no special coordination or behavior as a whole. Within that view, there is little to be learnt from studying the process. It is just number crunching, without new insight.

But this view is, I believe, mistaken. The movement forward of the adapting cells, and the progressive adaptations that take place as each part rubs up to its neighbors, shapes them, and is shaped by them, coordinates the whole. Some profound coordination of the whole is occurring. I do not think this coordination is "merely" the effect of multiple random events and effects. There are strong reasons to think that this aggregation of apparently random events is, instead, a very highly organized larger structure-preserving process, in which the process in the large, does progressively pay attention to the whole, reflect the whole, and extend and make more beautiful the whole.

To avoid confusing the reader, I must enter a clarification here. Throughout the four volumes of *The Nature of Order*, I have used the phrase "structure-preserving" to describe the transformations which move a system forward in time.[11] However, as a result of many discussions, I have concluded that possibly the phrase "structure-preserving" is not quite right. What I am referring to is a structure-confirming, structure-enhancing, structure-extending, structure-strengthening, structure-sensitive process. Mere "preserving" sounds rather too static and simplistic for the more active unfolding that is actually going on. We might better call them "*wholeness-extending*", and I have chosen to use this improved terminology throughout this paper.

That is what makes it worth studying. It is a kind of computation, entirely unfamiliar to conventional mathematics, but a computation nonetheless, and one that reaches profound results. By observing this kind of computation going on, and then, hopefully understanding it well enough to simulate it, we may lead to a new era of our ability to think.

In this paper, I rely heavily on examples. This is partly because the subject of harmony-seeking computations is difficult, and one builds a sense of its feasibility by considering many kinds of examples from different spheres, and slowly grasping the general propositions that underlie all of them. It is also because harmony-seeking computations occur in nature, and can also occur in human creative processes. The constant awareness of these two very different spheres, and the process of comparing them, is what, above all, makes this kind of computation interesting.

HARMONY-SEEKING COMPUTATIONS

Relation of a Given Computation to the Larger Whole Beyond It

In the examples I give, the essential thing, an essential feature of the process described, is that the new configuration that is created does something to participate in, strengthen, extend, and enlarge the force or "presence", of some larger configuration that stands in the world around it. In many cases, this larger configuration is not itself amenable to change during the "computation", but it does nevertheless enter in as a crucial factor.

If a lane exists, connecting two villages, and the road between them is then asphalted one day, the new wider and harder road is primarily affecting the configuration of these two villages, a tract of land much larger than the lane itself. Indeed, the new road may (according to its position in the region) also affect and strengthen an even larger part of the region's economic network. We recognize this as a fundamentally useful act.

A played musical note is most effective according to the degree that it develops a melodic line, a rhythm, or a harmony. If it does all three simultaneously, it will be most effective of all. Once again, we recognize the usefulness of this act.

These are simple cases. Other cases involve creation of the most highly organized and complex wholes. These have, so far, eluded us in science. Even the exciting and useful theories of bifurcation, symmetry breaking, chaos, and generative algorithms, have not yet deeply plumbed the meaning, or the origin, of true complexity.

All this is obvious, hardly more than the most ordinary common sense. Yet, obvious or not, the extraordinary fact remains that this kind of adaptive process (the farmer building his fence with respect for the land) does not currently have an acknowledged part to play in present day theories of algorithms, in developmental biology, in architecture, nor even in system theory. It is just not part of the mental models in our tool bag that we currently address and use.

Once we take this seriously, and add knowledge and perception of this type of transformation to our kit of tools, it will be natural to contemplate the possibility of a kind of computation that examines possible moves forward from a given state of a system, tests them, and evaluates the extent to which each of these moves participates in and adds to the larger wholes in the world around it, then chooses the most effective, the most harmony creating, the most successfully whole-creating! Such a process of computing, if it can be attained, would be enormously powerful, and powerful in its implications.

The Essence Of Harmony-Seeking Computation

The essence of the harmony-seeking computation lies in the following. It creates new configurations, unknown configurations, and *good* ones, by taking off from a known configuration, but without (necessarily) requiring the input of human creativity. The process *itself* is creative.

Consider a given configuration with certain features that are visible, and which, in the main, define whatever whole, or wholeness, we see in the configuration. But, in addition, in every configuration there are also traces, hints, of dim structures, not yet fully developed, but existing in a latent form, "between the lines" of the configuration. What happens in harmony-seeking computation is that some process "notices" these latent structures, and enhances them, develops them. Sometimes what develops may be relatively modest with respect to the size of the entire configuration. At other times, very, very large structures may also be latent in a configuration, and these large structures are enhanced. If the whole-seeking computation identifies this latent larger whole, and strengthens it, so that what was before only barely visible, now becomes strong and easily visible, the configuration will seem, to an untrained eye, to have gone in a new direction all by itself. It is this process that is the essence of all harmony-seeking computation.

Example 1: Embryogenesis

Consider an example of embryogenesis, a growing mouse foot. Here is how it grows in four days, from the 12th day to the 15th day. Each stage contains within it some structure that is defined, and some that is for the time being a vague and fuzzy mass of jelly, which anticipates the shape of the next step, which then consolidates and solidifies what was merely latent only hours before.

Figure 3 Step by step development of a mouse foot

What are some of the transformations that constitute the wholeness-preserving character of these moves? The form is governed by an axis from the attachment to the body to the tip. In the second slide we see the emergence of a STRONG CENTER at the tip itself, forming a thick BOUNDARY (in one dimension) to the arm, together with a dark fork that has emerged within the jelly. This center is then accentuated by the appearance of a GRADIENT leading to the fingers, and this gradient is then embodied in the 3rd and 4th slides by LEVELS OF SCALE, CONTRAST, and LOCAL SYMMETRIES, and finally finding expression in the GOOD SHAPE of the whole.

Example 2: A Bench Around A Tree

Consider the example of a growing willow tree, and the act of the landowner who chooses to build a bench around the base of the trunk. The bench places a BOUNDARY around the tree. Compared to the size of the trunk, this is a tiny act, a tiny step. But the bench starts from the cylinder of the three trunks, puts a small ring-shaped structure around it (the seat). We may say that the possibility of this seat was inherent in the previous structure, latent there, and that the bench builder simply made explicit and more solid the structure that was present in a weak and latent form, at the base of the willow tree, already.

Figures 4A, 4B and 4C A delightful seat around a tree, being woven in basket form and then finished with turf
Figure 4D The finished seat

I mean this literally, not metaphorically. The ring-shaped structure that later finds embodiment and physical form in the seat, is already present before the seat is built, in the ring-shaped system of symmetries and sub-symmetries of the space around the tree, because they are induced by the presence of the trunk and its roughly cylindrical shape. This statement is the crux, and in this statement – when it is generalized – lies the mathematical kernel of what I have to say in this paper.

In each of these two examples, the mouse foot, and the bench around the willow tree, these steps build on the structure that is *there* before; they do not destroy it or interfere with it, but rather enhance it, elaborate it, deepen it. As a result, what arises has wholeness, coherence, and beauty. *That is the trick, in a nutshell.* By continuously preserving and enhancing the existing structure, a beautiful thing arises, naturally. Yet, because each whole is unique, and the idiosyncrasies lying latent in it are also unique, the new whole that springs from this process is unpredictable, original, and creative.[12]

To perform this trick we must ask rather more precisely what it *means* to take steps that intensify the latent structure that is present in a configuration. Can we define this idea precisely enough, so that we can start doing it on purpose?

STRUCTURE OF WHOLENESS

The idea that every given geometric configuration might have its own "wholeness" seems odd at first. However, there is a real and definable mathematical structure underlying any configuration that captures, or "is", the structure of the whole configuration, its *global* structure. Over the course of years I have begun to call this structure *the wholeness*. It is not uniquely defined (mainly because the structure is so difficult and complex to describe in precise detail), but it is definite, and various attempts to represent this structure approximately, if they are any good, all have something in common. There is a deeper, global "something" there.[13]

I first began to recognize this possibility in experiments undertaken at the Harvard Center for Cognitive Studies in the early 1960s (described above, in the preface). These experiments were inspired by the early 20th-century gestalt psychologists – Wertheimer, Koehler, Koffka and others – who wrote extensively about the perception and cognition of wholeness and of wholes in the world around us.[14] In the early 1960s, Huggins and I used various experimental techniques to find out how different perceivers see configurations.[15] The essence of the technique was to ask people to group different patterns according to their degrees of similarity. It became clear that there are broadly two kinds of perceivers, those who see patterns holistically, and those who see the patterns according to analytical classifications. The perceivers who saw analytically were in the majority, about 85%. The perceivers who saw holistically were in the minority – about 15%; these perceivers grouped patterns according to their overall figural character.

However, as explained on pages 3-6 of the prefatory remarks, it was clear that the figural perception was somehow more real, while the analytical perception was arbitrary, and made according to arbitrary invented schemes. The figural quality was inherent in the material. The ability to perceive figurally was therefore of enormous importance.

I discovered, later, that it is possible to train people to perceive figurally; but it takes a good deal of effort to overcome what I believe is a culturally induced tendency (especially among academic people) to be analytical – and thus perhaps understandable among Radcliffe students.

In broad terms, we may say that the global configuration is a structure of the main features of the configuration – in particular *when it is seen as a whole*.[16] During the same series of experiments, I made a significant discovery. The coherence of the different configurations could be measured by counting the number of locally symmetric sub-configurations it contains.[17] This measure gives results in strong agreement with experimental results from cognition, memory, perception and so on. Thus a configuration and its structure do depend, in some considerable degree, on the nesting of LOCAL SYMMETRIES and on the structure of the system of locally symmetric subsets that occur in it. The fact that a simple measure (the number of locally symmetric subsets) to some extent characterizes the different configurations, lends further weight to the idea that for a given configuration, *the* wholeness is something real.

EXAMPLES OF HARMONY-SEEKING COMPUTATIONS IN DIFFERENT DISCIPLINES

Example 3: Hayricks in a Field

The photograph shows two modest hayricks placed in a rolling field in Romania[18]. The placement is done in such a way as to complement the land, to enrich the land; it is humble, self effacing, there is no ego visible, only concern that the land and its harmony should be enlarged.

Figure 5 Hay ricks in Romania

How is one to characterize what is happening here? What is it, that we see, and recognize as so supremely successful? Of course, the placement is primarily practical. The hay ricks are placed on almost-level spots, but ones which have a very slight slope so that they can drain. But there are many subtle harmonies, also visible. The ricks are on the same scale as the trees on the ridge; the trees gently surround and enclose the place where the ricks are; the rolling quality of the land, with its dips and hollows, and the hedges and trees as they are placed, is respected. The two hayricks are placed with considerable care, on slight outcrops of land, so that the hayrick is not only kept dry, and free from rot, but it actively enhances and emphasizes the slight bumps in the land. In addition, the actual structure of the hayricks themselves (each rick a tree of light timber, with hay piled up on it), mirrors and echoes the character of the grass land and of the surrounding vegetation.

Below, I show a sketch, which in very broad-brush terms, approximates the wholeness present in that place.

Figure 6 Impressionistic depiction of the wholeness structure in the land

The wholeness includes the sinuous curves, the kinds of shapes which are present in the field, the dips and hollows, the surrounding trees, the trees along the ridge, and the fallen down enclosing fence below. I believe that the people who tended the land, placed the trees forming a hedge on the ridge, allowed the hollows in the land, and who built and placed these hayricks were, consciously or unconsciously, performing a harmony-seeking computations every time they worked the land. There are ECHOES of shape and size between land and hayricks, ECHOES of a certain kind of curve, LOCAL SYMMETRIES in the ricks themselves, the placing of the ricks emphasizes naturally occurring STRONG CENTERS that are generated by shelves and flattened places, bounded so that the hayricks nestled in the land, are subdued and congruent, and inside the structure which exists. The hayricks are kindly to the land, but they are placed with enormous care. They follow the wholeness of the land itself. The ricks fit into the land because of the way they function and because of what they are made of. In order to function well, they have to sit on a locally flat surface that is uphill from where water collects, and downhill from the crest of the hills so that the effort it takes to stack the hay is minimized. They are made of hay, the material produced by the place, so their scale, color and form are the same as their surroundings. Nothing foreign is introduced into the place for the sake of function alone. Form and function together are part of the wholeness of the place. The wholeness is a harmonious structural and geometric quality within the whole.

In the examples that follow, I begin describing, structurally, what is it that has taken place to generate such harmony, and what it is in other cases that has caused the breakdown of harmony.

Example 4: Giant Wind Turbines on the Danish Coast.

Let us look at what 20^{th}- and 21^{st}-century thinking about engineering and design often does. The thinking is rationalized, usually, by more or less algorithmic formulations of various kinds. However, when these computations about subtle and delicate wholes are rationalized according to current ways of thinking, they usually go wrong as far as respect for the larger underlying background structure is concerned. The subtlety and depth of the existing global structure in a landscape is not so easily recognized by technological thinking. For example, in the photograph, the land has a global wholeness that is made of layered, nested flat plates, a two dimensional system of plates. However, the ecologist who placed the two giant turbines was evidently unconcerned about creating harmony with this structure.

Figure 7 Wind turbines on the flat north coast of Denmark

Figure 8A Diagram of the wholeness before construction of the turbines

Figure 8B Diagram of the wholeness after construction of the turbines

Cover up the two towers on the photograph, so that you can see the land as it was before they were placed there. It is hard to see it, because the towers are so obtrusive, and so alien to the kind of structure that is there. When an ecologist becomes too concerned with one aspect of sustainability (the importance of wind energy, as in this case), and then plants 300 foot high windmills on an ocean front, destroying the landscape in the process – this is no longer a W-E-transformation, but rather a well-meaning but ill-judged algorithmic computation of some inappropriate kind about cost benefit. In this case, the wholeness and the structure of the place have been severely damaged. Sometimes, the place, its landscape and its internal adaptations – often the result of centuries of patient work – are irredeemably destroyed.

In the coastal landscape where the Danish turbines have been placed, the structure that was there before is flat like a disc, a great flat disc, the size of the bay, flatted at the edge where the waves meet the sand and the grassland just beyond it. This enormous gentle discus-shape, perhaps half a mile across, is suddenly violated by the two vertical structures, sprouting turbine blades dumped on it. The two massive vertical things have no relation to the structure of the subtle disc that lay in the land before. The slash of the new straight road is at odds with the curves of the land. And, in the foreground there are two teeny little sheds that look lost and forlorn, too. The building site is not even harmonious with itself. Altogether, the structure has been violated.

Compare what is happening in the hayrick example, and in the turbine example. In the case of the hayricks, the process of placing them has left the previously existing structure alone, indeed the process enhances it, strengthens it, increase its harmony. But in the case of the turbines, their shape and placement are at odds with the previously existing structure, they damage it, they destroy it.

What is the "it" that is being left alone in the first case and is being damaged and destroyed in the second case? It is the *wholeness*, that system of centers existing in the geometry of each place, which give it its character, shape, organization, rhythm, feeling, and global configuration and relation to the land around it. One may describe the relation of the action to the wholeness like this: in the first case, the injection is friendly, helpful: what is there is healed, complemented, and continued; in the second case, the injection is violent, disrespectful, and structurally at war with the land that was there before.

My judgment about these two injections is not a romantic yearning for an idyllic past, but a structural judgment about something that is objectively present in one case, and missing in the other. It is also true that wholeness-enhancing processes allow growth and extension to continue while the others freeze things and inhibit improvement. One injection respects what is there, the other does not. And these structural judgments are about phenomena that are essentially mathematical in character. It is not just a question of feeling, but also a question of structural congruence, or the lack of it, that exists between an existing structure and an injected structure which has been brought into it.

We very much need to understand more about this kind of structural respect.

In order to understand the idea of harmony-seeking computations better, it is useful to examine a wide series of examples of such computations, most of them occurring in the real physical world. We may also call them whole-seeking computations, or healing computations. The following examples are taken from the fields of biology, architecture, ecology, embryology, decorative arts, town planning, entomology, symmetry breaking, and astrophysics.

Example 5. Evolution of the whorled cap of *Acetabularia* – 6 steps

This example is taken from Brian Goodwin's work on the morphogenesis of *Acetabularia*, a single-celled alga which takes the form of a stem with a cupped whorl at the end. Goodwin has studied the growth of this form, and the series of transformations that generate the form of the completed alga.[19]

Figure 9 Whorls forming at the tips of developing Acetabularia

Figure 10 Goodwin's five diagram sequence

The key morphogenetic sequence that Goodwin describes are illustrated in his five-diagram sequence above, showing the stem bulging and then making the whorl. It goes as follows:

1 to 2. A hemisphere, formed by the bulging upward growth of the cell $\xrightarrow{\text{turns into}}$ a hemisphere with a small centrally symmetric swelling on top (an ogee curve).

2 to 3.The hemisphere with a small centrally symmetric swelling on top $\xrightarrow{\text{turns into}}$ an elongation of the swelling forming a neck.

3 to 4. The hemisphere with the elongated narrow neck $\xrightarrow{\text{turns into}}$ a neck with a flattened top, upper end.

4 to 5. The flattened neck on top of the hemisphere, grows a ring of small projections (a whorl) that ultimately turn into the umbrella shape visible in the completed algae.

5 to 6. The whorl formed around the top of the cylinder grows outward $\xrightarrow{\text{now turns into}}$ a full umbrella-like ring of small projections that ultimately turn into the umbrella shaped cap visible in the completed algae.

As Goodwin says, strongly and repeatedly in his writings on the *Acetabularia* example, this growth sequence is generated by natural physical phenomena. It has relatively little to do with DNA or genetic guidance. What is happening is a naturally occurring and inevitable progression of morphological transformations that arise directly from the geometry and dynamics of the form itself, to generate the next geometry.[20]

It is highly significant that a prominent biologist has begun taking the emphasis of morphogenesis away from the influence of DNA, and has begun to see the morphogenesis as an autonomous process in the geometry itself.

There is an interesting sequel to this story. When Brian first showed me the diagrammatic sequence illustrated above, I asked him if the progression from round-ended neck to flat-topped neck was correct. Why do you ask? he said. I said that, looking at this purely from the point of view of structure-preserving transformations, I would have expected something different.

It seemed to me very unlikely that a round topped structure could transform into a flat-topped structure. There is nothing like the flat top latent in the configuration. There is a *ring* latent in a hemispherical configuration, but further down the curve. If you have a rounded hill, or hemispherical end of a neck-like structure, there is, inherent in that configuration, a latent structure something like a shoulder, which is incipient near the top of the hill and just below it (see left-hand sketch below). In the right-hand photo below, we see such a structure on the prehistoric Wittenham Clumps near Oxford. It evidently was, for the builders who made these mounds, a natural way to continue the smooth structure of a rounded hill. Somehow this point of inflexion in the curve of the hill is a natural development from the smooth curve.

Figures 11A and 11B   One of the Wittenham clumps near Oxford and A diagram of the structure created on the mound by the prehistoric embankment

If you are in doubt that it is "natural", consider two possible transformations of a hemispherical hill. *A* just flattens the top by squashing it into a flat surface. *B* introduces a band below the top, with two points of inflection in the curve, as shown below. Ask yourself which of these leaves the hill alone more, and preserves the global structure of the hill better. I think that you will agree that it is *B*. It is in this sense that *B* is more natural than *A*.

So I said to Brian, referring to his Acetabularia diagrams: If such a configuration were latent, and did then transform to create such a ring-like shoulder, then that *shoulder*, not the top, would be the most likely place where whorls and other irregularities might naturally form. So I asked him again, Are you sure that the real morphogenesis of *Acetabularia* doesn't go like that, rather than the way your diagram portrays it? Like this:

Figure 12   My analysis of the developmental sequence, based on W-E-transformations

Brian laughed, and told me: You are absolutely right: that *is* what actually happens – I just got the diagram wrong.

This story sheds interesting light on the way sophisticated harmony-seeking computations may be able to help both observational biology, and theoretical biology, and structural modeling and simulation. We come back to this later. The important point for now is that it is possible to make accurate statements about what structures are latent at a given phase of morphogenesis; and it is therefore possible to say, objectively, how things are likely to unfold.

Look again at the photograph of the *Acetabularia*, and look inside the caps: there is indeed a remaining *hillock*, not a flat, just as my whole-seeking computation predicted. At the time I made the prediction, I had never seen enough detail of *Acetabularia* to know about this kind of cross-section. My guess arose solely from wholeness considerations, and from thought about what would be structure-preserving.

Figure 13   Enlarged photograph of Acetabularia showing hillocks, not flats, at the center of the whorls

Example 6. Two possible plans for a five–story apartment building in Tokyo

Figure 14A and 14B Two possible plans for the five story Emoto apartment building in Tokyo, South is at 10.30 pm.

Figure 14A shows a plan for this apartment building in Tokyo, done by a student. Figure 14B plan shows the five-story apartment building that I subsequently built on the site.[21] Though typical of many architectural projects in 1987 (the era of the project), the student's plan plainly lacks the wholeness-preserving quality of the final plan. The layout process for the final design has about fifty structure-preserving steps, described in detail in the reference.[22]

The plan as built complements and intensifies the wholeness of the site. This is clear from just looking at the ground plan alone. However, to emphasize what this really means, and to illustrate the deeper structures that are involved, I describe some specific features of the global structure that existed in that place, and that were solidified and complemented by the building placed there. Some of the structural features of the original site are: The center formed by the Y-configuration of the fork; The curving nature of the two branches; The space between the forks, and its specific V-shape; The southern spot hit by the sun.

The new building, when injected into this site, does the following things that help to accentuate and strengthen these structural features of the place:

- The sharp end is given a snub nose, creating a place in front of it (it became the entry to a shop).
- The front gardens of ground floor apartments are placed to form a boundary between the building and the street.
- Since the boundary is of uniform width, the space of the street is maintained and continues its positive shape.
- The building is wrapped around the sunny south-facing spot on the site, making sunny space for all the apartment owners.
- On upper floors an inner layer of galleries and terraces form a boundary to this place.

As these various features are consolidated, they form a coherent structure, in which each feature supports and helps the others. Marked among this coherence is the presence of positive space *throughout*, even in a geometrically complex configuration; the fact that there are levels of scale within the structure; the focus of major centers to the structure, each with its own strength and beauty.

Figure 15 The completed Emoto apartment building

The process of building, and injecting, this structure into the configuration of the site, and designing and planning and building the entire five story-complex, I view *altogether* as a complex computation. It takes the initial configuration (the site) as its starting point, then transforms the configuration, by about 50 steps, finally ending with the configuration illustrated. Although the work of an architect of this kind would be conventionally viewed as an artistic act, hence as an arbitrary and "private" process, I believe this to be seriously in error. In the process illustrated here, each step is based on the configuration that is there at the moment before, and the latent structures that exist, and then makes one step forward that specifically, and carefully, strengthens or supports, *and evolves naturally out of*, the structure that was there before.

Example 7. An ornament drawn by Hiro Nakano – 6 steps

On the next page there is a much simpler example – just the evolution of a sketch – but following the same kind of process. The sketch starts with a row of evenly spaced dots. Then, it goes like this:

- Make a circle around alternate dots. (ALTERNATING REPETITION)
- Intensify these new centers with an additional circle, and also make a much smaller circle around every other dot. (STRONG CENTERS, ALTERNATING REPETITION)
- Take the bigger circles and subdivide each into eight sectors, four plain, and four that are fan-shaped alternating with the plain ones. Give each fan-shaped sector a special shape to emphasize it and give it more detail. (GOOD SHAPE, LEVELS OF SCALE, ALTERNATING REPETITION, BOUNDARIES).
- Add a strong dark, shaded band, to connect the larger circles with the smaller ones they alternate with more strongly. (CONTRAST, NOT SEPARATENESS, LOCAL SYMMMETRIES).
- Add two additional lines, making borders to frame the dark band, and thus creating a still stronger sense of unity. (BOUNDARIES, NOT SEPARATENESS).

- Darken the central band much more, and blacken the flower shaped quadrants of the inner large circles. (CONTRAST).

In the sketch, through this very simple sequence of six steps, we see the growth of a coherent and sophisticated structure, from extraordinarily simple moves. The names given in parentheses at the end of each step (above), are references to the 15 properties and transformations, defined in *The Nature of Order*.[23] As we see later, these properties and transformations play a fundamental role in the creation of all wholeness.

Figure 16 Six W-E-transformations leading to the creation of a new ornament

This example is instructive because it is complex enough to be interesting; we get a vision, for the first time among these examples, how recursive use of these transformations on the emerging centers, and on the centers that the transformations themselves generate, is potentially a complete and very powerful process, perhaps one capable of generating all complex configurations in their totality.

However, there are still many years of work needed to generalize this harmony-seeking computation process in an effective and operational form for the broad range of problems we may wish to apply it to.

Example 8. Historical evolution of St Mark's Square – 10 cycles

Figure 17 Plan of St Mark's Square

Figure 17 shows the plan of St. Mark's Square, Venice, one of the most famous and beautiful public places in the world. This marvelous and highly complex structure grew, steadily, over a period of about a thousand years. What happened there may be best understood as a series of harmony-seeking computations, carried out at intervals, with intense and deliberate care. Although the series of growth cycles shown below is historically accurate, I cannot be certain that the nature of the activity undertaken at each stage was exactly what I have indicated. However, I am fairly certain that it must have gone roughly as I suggest: the fact that it is possible to define a coherent and rather simple paradigm for each cycle of activity, and that in their essentials they are all the same, is remarkable, and strongly suggests that what happened was indeed something along the lines of the underlying computation I have proposed.[24]

Figures 18A,B,C,D,E,F,G,H,I,J Ten cycles of W-E-transformations in the history of 1000 year development of St Mark's Square

In every cycle, there are three entities at work:

1. *The configuration in the large*, roughly the size of the whole of the frame, although focus of attention is usually on an area about half the size of the illustrated frame – shifting from cycle to cycle.
2. *The particular latent center*, shown pink, which is the latent center that most calls for elaboration and development. The area of the latent center is usually slightly larger than the pink area: the pink shows its focus.
3. *One or more smaller centers*, shown blue area, that are built in order to strengthen and make more forceful the presently weak latent center. Placing these blue centers is always done in such a way to strengthen both the previously latent center, and the large configuration that is helped as a structure by the elaboration and articulation of the pink.

The procedure goes like this: Find the latent center that is most salient, that seems most likely to strengthen the wholeness of the larger configuration. Act locally, in such a way that this latent center gets strengthened, and so that this strengthening helps, also, to strengthen the largest whole. Repeat this cycle ten times over a period of about 1000 years, (roughly once per century), and the result was St Mark's Square as we know it today. This rule explains (or generates) the ten actual cycles of construction and improvement which occurred around St. Mark's from 600AD to 1600AD.

Example 9. Formation of Giant Voids in the Universe: a Very Large Example of a Generated Wholeness

In recent years, attention of cosmologists has focused on the truly enormous structures of galaxies and galactic clusters in the universe, now called filaments. Subsequently, even greater attention has been focused on the voids that seem to be encircled by these filaments. The filaments are typically hundreds of millions of light years long, and tens of million of light years thick. The voids are often hundreds of millions of light years in diameter.

Figure 19 Shechtman et al, the Las Campanas Redshift Survey,1996, showing voids and filaments

The image above is one of the early pictures of the distribution of galaxies and galaxy clusters that for the first time clearly showed us that these voids and filaments existed.[25] One of the remarkable things about the voids is that they appear to be convex, and quasi-spherical. To me that implies the presence of some active principle pushing outwards. In terms of the language of wholeness, the existence of THE VOID (a general structure existing at all scales, and not specific to astrophysics and cosmology), and of thick BOUNDARIES, and of the scale ratio given by LEVELS OF SCALE, is to be expected in all wholeness, since these are three of the 15 main ways in which coherent structure typically occurs. Further, the astrophysical voids seem to be close packed, rather like soap bubbles, and, like soap bubbles, where the bubbles meet, there is a film where the boundary material is concentrated. In just this way, the galaxies are spread out in linear filaments.

Figure 20 Artist's sketch of the 3D structure, showing roughly how the ring-like filaments lie in space, on the surface of the voids

The so called Great Wall is 500 Mpc long, 200 Mpc wide, and 15 light-years thick. (1500 million light-years long, 600 million light-years across (band) and 15 million light-years thick. Filaments are typically 70 to 150 Mpc in length. (200 to 500 million light-years). Voids are 10 to 150 Mpc (30 to 500 million light-years in diameter). The actual physical size of one of these voids, is unimaginably huge, about 100 million light years. It is in miles, $186{,}000 * 3600 * 365 * 24 * 100{,}000{,}000 = 5865696 * 10^6 * 10^8 \approx 6 * 10^6 * 10^6 * 10^8 = 6 * 10^{20}$ miles. A jet plane flying at 600 mph, would take 10^{14} years to cross this void – something like 5000 times the age of the universe itself. I say this to emphasize the huge size of the voids that we are talking about, and in particular to draw attention to the fact that if the void is that huge, the ring thickness could in principle be almost anything, and the ratio of ring thickness to ring diameter could (hypothetically) have a large range of possible values.

Yet that is not what we observe. One of the structure's most unexpected features is the ratio of filament thickness to void diameter, about 1 to 10, roughly one order of magnitude. Why is this interesting? Because the filaments could have been much thinner compared with the diameter of the voids, or could have been fatter compared with the voids. There is no *a priori* reason for the ratio to be in the range of 1:10 (one order of magnitude). It could just as easily be 100 or 1000 or 1,000,000 – that is, 2 or 3 or 6 orders of magnitude. Why is the ratio of filament thickness to void diameter on the order of one order of magnitude? The next picture, a simulation by George Lake and Tom Quinn, shows this structure more clearly.[26]

Figure 21 The Lake-Quinn simulation

Stated differently, what possible reason could there be that the rings around such a colossally enormous, almost impossibly huge-to-our-understanding, structure in the universe should have roughly the same geometric proportion as a gold wedding ring or as a rubber band? Is this not pushing coincidence? In the absence of any current theory that might give us a predictive model of the relative size (unless one fits the parameters just to get the result we observe), it is worth speculating whether the universe itself – its action – is (among other things) making harmony-seeking computations, and that the result of these computations is the system of voids and filaments we see, because it so closely parallels our own cognitive and intuitive sense of the relative dimension of a *typical* ring's thickness and the diameters of the space that it surrounds.

What exactly do I mean by this? I mean that there is a certain class of mathematical structures, in space itself, that has features that, for spatial and computational reasons, gradually appear in space – because of the geometry of space itself, not because of what we know as forces.

This hypothesis provides us with something very much *like* teleology, *without being teleology at all*. It strongly indicates that there is inherent, in space itself, a motion caused by the whole that can be understood in mathematical terms that are in principle quite simple. Even very small modifications of space cause a reshuffling, or reorganization, of the entire system of local symmetries, sometimes extending far beyond the point where the disturbance occurred. This new global configuration introduces new latent centers that were not present before, sometimes far from the original disturbance. If harmony-seeking computations then draw out and strengthen the latent centers that were at first only dimly present, we then see how space itself, by virtue of its geometric organization, can call out, or pull out, new configurations.

I say that this process is “like” teleology, because it provides the system with a constantly moving goal that forms new structures all the time. It does not come merely from the dynamic trajectories of the past. At the same time, it is not at all like the teleology that scientists quite properly reject, because the orientation or arrow of the system’s next move comes from the current state of system itself, not from a future or external hand. It is an internally fueled trajectory, caused by the phenomenon of the geometrical whole: precisely that thing that has so far escaped us.

Next time you are in the bath, try stirring the foam that lies on the surface of the bathwater. Very often you will see that the foam moves in ringlike swirls on the surface of the water, and once again the roughly circular swirling ring-shapes that are formed have a diameter to thickness ratio in the range of about one order of magnitude. I do not believe the ring ratio merely fits a cognitive niche. Rather, I suspect that there are, deep in geometry of space, reasons why ring-like structures with this kind of ratio are likely to occur. In current jargon, rings of this particular ratio might be viewed as attractors in some phase space. However, the discovery of geometric attractors in the solutions to systems of dynamic equations is, in my view, only one particular manifestation of the far more general harmony-seeking computations that occur naturally in three-dimensional space. The tendency for such structures to occur might be shared over a very wide range of circumstances, and the voids and ring-filaments in the cosmos might form in shapes similar to the foam in the bath, not for dynamic reasons having to do with physics, but for mathematical reasons having to do with the structure of space itself.

But if we accept that the harmony-seeking computation is based on transformations that are (for whatever reason) largely congruent with human cognition and mentality, then we may not be silly to consider such a process as an explanatory process even in the most outlandish structures.

This supposes then that there may be mathematical reasons that generate circles or ring-like filaments whose thickness is just about one order of magnitude smaller than their diameter. There is, at the moment, no astrophysical theory that would predict this result – at least none that I know of – but, from the point of view of harmony-seeking computation, there is a possibility that the cognitive simplicity of about this ratio for a ring-structure could suggest that it might *have* to be so for mathematical or quasi-mathematical reasons.

Is this strange? Yes indeed, to our ears it is very strange. Yet is it more strange than Pauli's exclusion principle, when that was first put forward? I do not think so. A purely geometric harmony-seeking computation might explain at least one aspect of the structure better than any current physical theory. Does it seem unlikely? Perhaps. But in the absence of better computations, I do not think it can be considered entirely useless. At the very least, some reason needs to be found why such processes, congruent with cognition, might occur in structures at the scale of the universe and at the scale of human beings, and why the ring-ratios of both tend to be the same.

WHOLENESS-EXTENDING TRANSFORMATIONS

Viewing The Previous Examples as Computations

The previous examples all show, or suggest, the possibility of some kind of harmony-seeking computations at work in a variety of systems. In all cases, there is *some* kind of wholeness-extending process at work, mainly geometric in nature, sometimes intellectually graspable, sometimes not so clear. Apparently, as a result of this wholeness-extending (or W-E-) process, at each step the generated configuration moves a given configuration towards a slightly more harmonious, more coherent, state than it was in before.

If we choose to do so, we may regard *all* these examples – physical processes, simulations, and actual paper and pencil exercises – as *computations*, each of which takes its system forward, step by step, from an existing state W_1 to W_2, W_3, … W_i , W_{i+1}, and so forth, increasing structure and harmony and wholeness in the system as it goes. These steps are carefully calculated, normal in many spheres of human experience and in many natural systems. Some of them are done by animal instinct; some by human instinct; some by adherence to a certain tribal sensibility; some by concern for ecology. Many occur naturally (without human intervention) as part of the behavior of some complex physical system.

In the various examples given in this paper, I have used the word *computation* with a variety of different, though related meanings:

- *Intuitive computations*, made intuitively, by human craftsmen: hayricks; the bench around the tree; the construction of St Mark's Square.
- *Conscious computations*, made consciously by a designer, according to a consciously used scheme that calls for certain actions to be performed

in sequence: the Emoto apartment building in Tokyo; the ornamental drawing by Nakano.

- *Natural computations*, possessing explanatory power for straightforward scientific problems that have not yet been solved: the development of the whorl tips of *Acetabularia*; geese flocking to form a V-shape.
- *As yet half-formulated harmony-seeking computations*, which may be implicit in the behavior of different complex systems, and which, if so, would shed important light on the nature of the phenomena: the formation of giant voids in the unfolding of the cosmos; the formation of blue sky and clouds with their respective complementary positive space; the growth of tree limbs in such a way as to create positive space between the limbs.

  In these cases we might also simply talk about the system behavior, and the rules according to which system behavior develops. It is perhaps odd to call them *computations*: but if we wish to simulate the system behavior that leads to harmonious results, we may need to use such a computation as the explanation of the physical system's behavior.

In a generic sense they are all computations. But in nearly all these cases they are not recognizable as computations in quite the way we presently understand calculations or algorithms. This does not make them trivial, it makes them fascinating. There are reasons to believe that a fundamental kind of process is at work in all these cases. If we can examine these computations, and begin to understand or extract the underlying way they work – all of them together – and if we succeed in getting the gist of this type of computation, we may find a way towards a powerful new way of computing that is guided by emerging harmony, and by a motion towards harmony.

So I view these examples as computations, because, after 20 years of work, it has slowly become clear that they all embody some kind of single process, or function, that has the capacity, starting with an arbitrary configuration, to reach a better and more harmonious configuration. Although the idea of a W-E-transformation is, for the moment, still somewhat loosely defined, there is a considerable degree of similarity between the different examples – in the way that they work and in what they accomplish. I conclude that it is reasonable to think of the examples as exemplars of the operation of a single type of mathematical function, or computation, at work.

To learn to see these examples as embodiments of a single type of computation, we must now work harder to be specific about what this deep seated type of computation might be, how it might work, and how, above all, we can put it to work, in cases where we seek better organization, better harmony, better deep adaptation and better coherence for some real-world configuration.

Experimental Confirmation

My own cognitive experiments show strong agreement among observers as to comparative judgments regarding what steps are wholeness-extending and what steps are not.[27] A step is a harmony-seeking step if it preserves or extends the wholeness structure. When asked to judge different possible steps, starting from a given configuration, in terms of the degree to which the steps are W-E (wholeness-extending) or not, people seem to agree – based on their intuitions, or on their own cognition. This alone strongly suggests that the quality of "being W-E" is to some degree objective. It gives a strong hint that the phenomenon reflects an underlying physical reality.

It is also true that if we ask people taking in part in such an experiment to make (in their own chosen terms) any kind of structural diagram in which they try to capture the wholeness of the thing they are looking at, before judging which steps forward from it are structure-preserving and which are not, the level of agreement between observers in judging what is W-E and what is not, goes up strongly. People who have performed such a private diagram-making process, even though their diagrams are not alike, then agree more strongly with one another about what steps forward from that structure are W-E. This occurs even though the making of a diagram that describes the structure is a private matter, and people are given no special instructions about how to do it, and the different diagrams people make are unlike. It seems to be enough that after simply *attempting* to represent the structure, they can then judge more reliably what is structure-preserving and what is not. This, once more, is evidence that the phenomenon of W-E is real and objective as a phenomenon, with objective structural content.

The W-E Postulate: Always Helping a Larger Whole to Form

How may we formulate, mathematically, the character of the wholeness-extending steps which occur in the examples?

In each case there is a whole, *W*, and within the whole a latent center that is being modified, transformed, shaped, or reshaped, by a certain step. This latent center is the focus of the transformation, and the latent center sets the boundary of the geometrical and physical transformations that are then actively being undertaken. Call this focal latent center *L*.

At the same time there is a larger whole, often an order of magnitude bigger than *L*. The transformation which is wholeness-extending, preserves the structure of *W*, and to do so modifies *L*, and modifies it in relation to the whole context around it. Thus the output from this step is a modification geometrically within *L*, but it is a function of both *L* and *W*. In addition, there is a sense in which *L* is being *fitted* to *W*, it is being made to fit *W*, to be congruent with *W*, adapted to *W*, harmonious with *W*. Further, in modifying *L* in this way, new centers are created within and around *L*. We refer to these new centers as N_i.

To undertake this transformation in such a way that it is indeed a wholeness-extending transformation, the various N_i are generated by 15 generic types of transformation acting on *L*. These are *the* 15 principal center-creating and structure-preserving transformations. These are enumerated and defined in considerable detail in chapter 2 of *The Nature of Order*, *Book 2 (The Process of Creating Life)*,[28] and in chapters 5 and 6 of *The Nature of Order, Book 1(The Phenomenon of Life)*.[29]

The W-E -Transformations o St Mark's Square

Now look back at figure 18. Here, in each cycle, the pink area is a latent center we may refer to as *L*, the next building to be built is built inside *L*, and the larger context of the whole St. Mark's area is *W*. However, there is now a subtlety. In a particular step, we know what *L* is, because we are looking back in time and see what the step was. But the people who actually did the step were not, at the time, so clear. They could not know what *L* was, until examination of the context and the larger whole revealed it to them. And there is a further subtlety. The context *W* is not something so vague and general as the whole St. Mark's area. It is, rather, a particular area within St. Mark's square, where the latent center *L* has been identified as being in need of improvement, or presenting itself for elaboration and strengthening. This latent center *L*, which plays a crucial role in the wholeness-extending transformations, *and* the detailed effects are created by the 15 transformations acting together. So it is actually the immediate local context of *L* that then gives rise to the step that transforms *L*.

Alternatively we might say that it is each latent center *L*, once identified, that is the entity to be transformed, and the transformation is to be done in such a way as to strengthen *L* in its ability to help and make cohesive the larger context *W*. The particular bits of building needed to carry out the transformation are the various buildings and partial buildings N_i that are generated by the action of the transformations.

Figure 22 A second look at one of the W-E-cycles in St Mark's Square

Here, for example, the red ellipse is *L*, the latent center formed by the three buildings around it. To confirm and strengthen *L*, the blue building mass is built, thus forming a stronger rectangular space, by enclosure, and establishing continuity with the buildings on the right of the latent configuration.

HARMONY-SEEKING RATHER THAN "EMERGENCE"

In recent years, conventional scientific wisdom has begun to see emergence of complex structure as a result of *coupled local atomic events generating larger wholes through interaction*. I do not think that this approach adequately accounts for the world's ability to generate highly ordered complex phenomena. Instead, I believe we must find a way of accounting for the specific generation of structure, in deeper ways than we have done so far. Concretely, I now compare the doctrine of emergence, with the richer notion that in many cases there is a *whole-based, harmony-seeking process that works by continually strengthening latent centers* and by operating on the wholeness that exists, to generate a new and deeper wholeness.

It has nowadays become almost commonplace to explain how geese fly in formation[30], how ants accomplish complex tasks together[31], or how slime mold (apparently an uncoordinated aggregate of cells) is able to move coherently as a whole[32]. In most cases, the explanation is, in some form or other, that the individual "cells" copy the behavior or action of their nearest neighbors; in many cases this ultra-simple rule of action explains considerably coherent movements of the larger body of cells.[33] In other more complicated cases, the atomic actions of individual cells are coupled, and the coupling helps more complex forms of behavior to occur in the group.[34] In either case, the resultant aggregate seems to be acting as a whole. This has become known as "emergence".

However, to be sure we are talking sense when we say these things, and to avoid exaggerating what we seem to have discovered under the rubric of emergence, it is important to analyze just what we mean by "acting as a whole" in these kinds of cases.

When health, or wholeness, or harmony, exists in a part of the world, what is under discussion is always the relationship of a given system to the larger world beyond that system. So the issue is not merely whether a group of elements act together. What is important is that when the elements are grouped together to form a system, the resulting system either does or does not act in such a way as to heal, or sustain, or improve, the coherence and health of the yet larger system around it, some part of the world *outside and beyond* the group, some system of which this group of elements is a part.

The *emergence* phenomenon is a *two*-fold relationship, between a set of elements and a group they form. The *harmony* phenomenon is a *three*-fold relationship, between a set of elements, the group they form, and the helpfulness of the ensuing group to the world beyond the group. Thus:

Emergence, a two level relationship **Harmony, a three level relationship**

Figure 23 Emergence, a two level relationship *Figure 24 Harmony, a three level relationship*

In the left hand diagram we have a group of elements, and because of their interactive coupling they act as a whole, represented by the outer circle. In the right hand diagram we also have a group of coupled elements that act as a whole because of the coupling, but in addition, *this whole acts to support or help some feature of the larger system*, represented by the outermost circle.

The system on the left is interested only in itself. It does nothing to help the world beyond it, and does not contribute harmony or health to the world beyond it. The system on the right is not only embedded in the bigger black ring: it *helps* that ring, and establishes harmony between the smaller ring of rings and the largest ring, so that the smaller ring is well adapted to the larger, and the large one benefits from the smaller.

What Does it Mean for One System to *Help* a Larger System?

Let us return to the example of St Mark's Square. At each cycle the process (through human agency) identifies a latent center in the larger configuration. This latent center is an area which is present, but weak, and which, if strengthened, would improve the coherence of the whole. The area immediately around that latent center is healed or made more whole by the injection of the repaired latent center.

Abstractly we may express this concept through the following diagram. The red area is a latent center, a weak center which has the capacity to heal the blue area beyond it: and it is able to do so by creation of the smaller white centers – they are built to create coherence in the red center. *However, the key point is that the healing will not take place, unless the red center then also helps to heal the larger blue center beyond it.*

Figure 25 Key condition for the healing to take place in a W-E-transformation

This is not really obscure. But as a key point, it is sometimes lost in the current flurry of attention that "emergence" nowadays draws unto itself.

Example 10. Flying Geese and the V-formation

Many boid simulations, due to Reynolds and others, essentially provide particle models in which the particles fly about, avoid each other, all go in the same direction, flock, play: altogether, simulate some of the behavior of birds in a fairly realistic fashion.[35] When used to model bird flock behavior in general, there is a surprising realism in some of these models. However, simulations of this type do not do as well when it comes to generating the characteristic V-formations of migrating Canada geese[36].

Much has been made of the fact that when in the V-formation, Canada geese increase their flying range by about 70%, because they use less energy, because of the vortex interactions. When aircraft arrange themselves in similar V-formations, a similar energy saving is experienced. But that does not explain how the V-shape *actually comes about* in the case of migrating geese.

I have looked at several boid simulations that demonstrate the flocking of birds. Seemingly random motions quickly give way to groups of birds that fly together and in the same direction, while maintaining a typical spacing distance from the other birds. *That does not in itself generate the V-formation*. Those cases that I have seen where a V-formation is claimed, or made to appear have all, in my experience, built this configuration implicitly within the rules that govern the birds motion.

Figure 26 The V-formation of Canada Geese Figure 27 Each bird flies in the outer wake of the bird ahead of it, since that is the spot which takes the least energy –just like riding a bike behind a bus, to catch a ride from the negative pressure in the airflow.

Let us consider what real geese may be doing. Reynolds' three basic boids rules (most frequently given) are these:

- If you are about to crash into another bird, turn around.
- If you are far away from other birds, head towards the nearest bird.
- Otherwise, fly in the same direction as the bird next to you.[37]

This system of three rules generates general flocking *very well*, in fact to an extent that seems amazing the first time you see it. But these three rules, alone, do not create a persistent, stable, *V-formation*. To do that we need to add two further rules:

- Try to fly in sweet spot of the *wake* of the nearest bird (which they can do because they can sense the change in pressure and the easier ride in that position). The optimum positions are in the outer wake, off-center from the axis -- not merely close to, or in the same direction as, the nearest bird.[38]
- If you are the leader and get tired (you are the only bird not getting the energy advantage), then drop away, and let another bird takes your place as leader.

There *is* a rule of follow the leader (though boid enthusiasts deny it), but the leader is not an arbitrary "king". Instead the wake rule means that birds follow one another, without electing a leader; but at any one time there is a temporary leader who gets defined by the fact that it is the only bird not behind another bird. Being in this position is tiring, and birds try to avoid it. So there is no permanently elected leader, but there is always a temporary leader, and that temporary leader keeps changing.

When we add these rules, two things happen. The dynamics do now generate a stable V, and the rules of action require that the birds compute or calculate in a way that makes a local center (the bird's own body) work to help a larger center (the V-formation as a whole), thus demonstrating the action of the basic rule I have mentioned earlier.

That can only be understood, and generated, by using a computation which looks at the growth of the whole, and the emerging structure of the V-configuration as a whole. It is likely, I think, that the real situation cannot *in principle* be modeled properly *without* a harmony-seeking transformation. That is because the real computation needed to generate the V *requires a computation which explicitly relates the individual to the whole*.

The current enthusiasm for “emergence” as evidenced in boid-like computer games and simulations does little to unravel the real and more subtle problems of the whole and its organization, which hinges (almost always) on the way that small parts *work to help a larger whole*, and the ways that the larger whole also shapes and modifies the action of the smaller parts.

The real issue is that the emergence that is being attributed to the birds (when they are viewed as mechanisms) is not as marvelously dumb or mechanistic as some mechanistic emergence enthusiasts like to think. The fact is that to produce the V-formation, the birds *themselves* have to perform a harmony-seeking computation, in the way that they act to relate themselves to the larger whole, and to help that larger whole. The fact that the birds themselves perform this harmony-seeking computation is the essence of the situation.

Example 11. Clouds and the Positive Space which Arises Between Them

In the following photographs we see various examples of cloudy skies. We are familiar with the shapes of clouds, and do not need to look at them just now. Instead look at the shape of the blue sky *between* the clouds. I have recently begun to study this blue sky carefully, and have began to notice (with something of the eye of a painter) that the blue patches are nearly always well-shaped. These are extremely different from the childlike cotton wool cloud formations we perhaps carry in our minds as a picture of the way clouds are (especially cumulus), where the clouds are objects and the sky is background.

Figure 28 Cotton wool clouds: the clouds are objects and the sky is background.
Figure 29 The real thing, a very different structure

Look at the real example in the photograph below.

Figure 30 Clouds and the sky between them, as they appear in nature

In every one of these cases, the blue sky is made of definite and positive shapes (POSITIVE SPACE). For a painter this notion is straightforward: a picture cannot be good unless *all* the spaces and components and fragments, have their own positive shape. But this positive space is quite tricky to formulate in mathematical terms. The idea of POSITIVE SPACE is something like convexity. In mathematics, a convex body is one which has the property that for any straight line that connects two points inside the body, all the remaining points in between, along that line, also lie entirely inside the body. Positive space is space that is, in shape, coherent, it is formed of positive, somewhat convex lumps that have definite and recognizable shape, but it is less tightly constrained than mathematical convexity – hence *quasi-convex*.

Figure 31 The blue systems are clearly visible here as having their own shape and character

There is no *a priori* reason to expect such POSITIVE SPACE to appear in the space between the clouds (seen in projection as blue sky). Yet, watching clouds blowing and changing minute by minute, they maintain the positive space of the blue, as well as the positive space of the white and gray clouds themselves, at each instant.

This may be expressed by a simple idea: the blue sky is not a mere background for the white clouds, but rather the space that we see as empty (or blue in projection) is itself an actual system, or dynamic object, acting on its own, and has its own shape and behavior, just as the white part (the cloud) does.

So how may we explain what nature is doing in these cases? Somehow the positiveness of space appears in the space between the clouds, but not for reasons obviously connected with the energetics of the system. Instead the system simply seems to have a disposition to have this positive space appear. What is causing it, and how does it work?

What we see in two dimensions in projection, is actually a three-dimensional phenomenon. The easiest way to imagine it, is to consider both the white bits, *and* the blue bits, as quasi-convex bodies. Somehow, as the clouds evolve, it is a co-evolving system in which this loose packing of differently sized blue and white quasi-convex bodies is maintained nearly all the time. Such a changing dynamic packing of irregular sized cells could be a more complex 3D analog of Taylor vortices or Benard convection. On this page I show two simulations from the laboratory of Professor Michael Cross at Caltech, demonstrating stages of Benard-like formations in a medium, one under the impetus of the Swift-Hohenberg equation, and one under the impetus of the complex Ginsburg-Landau equation.[39] They are far more regular that clouds. Still, one can see how the positive space could come about as a result of more complex interactions stemming from these kinds of effects, and possibly driven, in addition, by some iterative rule similar to the POSITIVE SPACE transformation.

Figure 32 Configuration arising from the Swift Hohenberg equation
Figure 33 Configuration arising from application of the complex Ginsburg Landau equation

Although this is not (in this form), as far as I know, part of the presently accepted physics of the cloud system, it is exactly what one would expect from a more accurate harmony-seeking computation, which has, as one of its most important structure-preserving transformations, the continuous maintenance of POSITIVE SPACE at every step, reflecting the fact that the space we see as blue sky between clouds, has its own rules of formation which are active systems maintaining their own coherence, and which thus shape the space of the cloud systems as much as being shaped by them.

Example 12 The Branching Shapes of Real Life Trees and the Positive Space between the Branches

The nature of positive space, as an active principle, which plays a major role in harmony-seeking computations, may be understood further by looking at the growth of trees. If we look at a tree we see volumes of positive space between the branches: it is visible, for example, in the left hand drawing below, which is a realistic drawing of a real tree. So, apparently, the empty space between the branches must, like the blue sky, have “energy”, a driving force for it to be a definite “thing”. How exactly might this be explained? The space between the branches has no obvious energy to push and create its positiveness. Nor does this idea yet have an accepted mathematical formulation in L-system theory, which focuses exclusively on the growth of branches, twigs and leaves.

Figure 34   A sketch of mine showing a real tree as it typically is in winter, and strongly showing the positive space generated between the tree's branches

Figure 35   On the right: a context-sensitive L-system simulation of tree growth in two adjacent, interacting trees. The spaces between branches still lack the intensity as centers caused by the action of each volume of space as a system in its own right

Prusinkiewicz has made great progress by simulating tree growth through the use of L-systems, and has recently developed "environmentally-sensitive" simulations, which create more realistic structure, which develops, in part, according to neighbouring conditions.[40] Yet all the L-system simulations that I have seen, including Figure 35, lack the positive character of the space between the branches shown in Figure 34. For example, see drawing on the right, above. [41]

One can readily account for the positive space phenomenon as it occurs in trees. The twigs and leaves in a growing tree are phototropic: they look for the light. Simultaneously, the tree also needs air and breeze, and the leaves and twigs and branches modify their growth to allow the passage of air, and to avoid crowding of leaves which deny them water and light. Thus, the "empty" volumes of space which nestle among the leaves are highly sophisticated systems, as necessary to the tree as the complementary organization of limbs and branches that provides structural strength and the flow of sap. As in all systems of positive space, the one system complements the other. Both are necessary to the tree's living character. In this case, observation of the harmony-seeking calculations performed by the action of the tree teaches us more about the tree as a system, and allows us, if we choose to do it, to write more convincing algorithms of a tree's morphogenesis.

I wonder if an extended new type of L-system might be constructed, which allows two parallel and complementary systems (1) the branch and leaf system and (2) the space volumes between the branches, to evolve and unfold in parallel, and in a way that makes each the inverse or geometric complement of the other?

WHOLENESS-EXTENDING TRANSFORMATIONS AND SYMMETRY BREAKING

The Possibility that Wholeness-Extending Transformations are Deep Generalizations of Symmetry-Breaking.

One of the most familiar analyses of the evolution of natural phenomena and configurations is the one that Ian Stewart and Martin Golubitsky have popularized: The phenomenon of symmetry breaking in *geometry*, not only in systems of equations and variables.[42]

Example 13. Dewdrops On A Spider's Thread or on a Stem

The explanation of symmetry breaking as an output from various natural phenomena, leading to interesting morphological results, is by now well known. The regular spacing of dewdrops on a spider's web is one case that Stewart and Golubitsky have discussed extensively. Crudely put, the water coated on a spider's web thread starts out as a uniformly coated cylinder of water, made roughly uniform in thickness by surface tension. When the surface tension starts to break up the continuity of the coating, it leads to a configuration that falls apart, but still repeats at least some of the symmetries present in the continuous cylinder, since there is no reason for all *those* symmetries to be removed as well.

This idea is *very* similar to the idea of a structure-preserving transformation. We have a structure: the infinite translational symmetries of the cylinder along the thread, and the rotational symmetries of the cylinder around the thread. As this system moves to a fragmented version (caused by the action of the surface tension, or by jiggling of the thread), the simplest end product is the configuration that destroys the least possible number of symmetries, or maintains as many of the remaining symmetries as possible. Golubitsky and Stewart's most recent work on symmetries in the equation systems of bifurcation theory, continue such ideas.[43]

But this one *particular* way of preserving the structure of what is there is very limited indeed, compared with *all* the possible ways of preserving and enhancing structure. In addition, even the symmetry-breaking interpretation of what is going on in the simple dewdrop example is geometrically too limited. Each dewdrop takes on local spherical symmetries and axial symmetries normal to the thread, and symmetries parallel to the thread but not aligned with it, that are not present in the symmetry scheme of the infinite cylinder.

Figure 36 Dewdrops on a spiders web

Figure 37 Dewdrops on a grass stem

In my view, the symmetry-breaking idea is not yet, by itself, sufficiently profound to be useful as a *general* theory explaining real-world complex configurations, or to account for harmony-seeking phenomenon. As I have said earlier, it has been postulated that "the" wholeness consists, in part, of the entire system of overlapping local symmetries at a wide variety of scales in a configuration.[44] We therefore need to have a view where somehow the underlying structure of *all* these symmetries, working together, is preserved. And further, the centers that are present in a given wholeness are not all LOCAL SYMMETRIES. Other centers are formed by GRADIENTS, ECHOES, THICK BOUNDARIES, DEEP INTERLOCK, POSITIVE SPACE, NOT SEPARATRENESS, and so on. These other properties and the entire structure, too, have to be preserved when a harmony-seeking computation starts with a currently existing structure, and finds its way to a stronger structure that is latent in this overall configuration, and can be brought out by a few transformations.

In summary, we may say: A wholeness-extending transformation is a more complex and richer version of the phenomenon whose simplest cases have in recent years been called "symmetry breaking" or "symmetry reduction". This nomenclature is rather over-simplified, and does not do justice to the real potential complexity of the underlying phenomenon. A W-E-transformation is a transformation that moves a complex configuration forward, retaining as much of its wholeness structure as possible, and drawing new structure from the latencies within the wholeness itself. In so doing the configuration usually becomes richer and more complex in unforeseeable ways that benefit the larger whole although this can happen without intervention by a decision-maker.

Local Symmetry Production

Indeed, even in the well-known cases of symmetry reduction, what we actually see, if we look closely, is symmetry *elaboration*. The infinite Euclidean 3D-space does indeed lose some global symmetries, that is part of what goes one. But at the same time, other, new local symmetries are generated or strengthened where none existed before. This is a better picture of how harmony-seeking computations work. The failure to see it, comes, I think, from the fact that in recent years the symmetry structure of the plane, or of the 3D-continuum, has been viewed too simply through the symmetries *of the whole*. It has not been sufficiently clear that there are, hidden in the plane, or in the continuum, an infinite number of *smaller local sets*, all over the place, some of which are themselves symmetrical, while others are not. There are, in the world, thus an infinity of systems of smaller and smaller symmetries that occur in these nested sets.[45] What happens when a whole evolves under harmony-seeking computations, is that many of these smaller symmetries are generated or strengthened where the latencies are strongest, thus gradually generating a hierarchical nesting of local symmetries at different levels of scale.

We may see the result of such a process in many famous buildings, for example in the Parthenon. The Parthenon is of course symmetrical in the large. That is obvious, and is these symmetries most people see when they first look at it. But when we examine all the locally symmetric sets, large, medium, and small, we find a truly astonishing number: the columns, the spaces between columns, the flutes of the column, the capitals, the metopes, the triglyphs, abacus, stylobate, entablature, guttae, steps, etc. The building is deceptively simple, yet the enormous number of locally symmetric sets in the configuration is the highest of all those measured by Salingaros among a considerable range of famous buildings. In general his findings are that the buildings with the most profound qualities, are those for which this density of local symmetries is highest.

Figure 38 Salingaros studied 25 famous buildings by counting the number and density of local symmetries, according to the measure proposed in NOO, also adding further elaborations to estimate the density of symmetries. He found that of the buildings he studied, the Parthenon had the highest count of local symmetries, and was highest in symmetry density.

This pervasive presence of many local symmetries, sometimes overlapping, is particularly visible in some cognitive experiments on black and white strips my colleagues and I did some years ago.[46] The left hand illustration shows the experimentally derived rank order of coherence, of thirty-five such strips, as measured by a variety of independent cognitive and perceptual tasks. Those at the top are (by a variety of experimental measures) found to be cognitively most coherent; those at the bottom least coherent. After two years puzzling over the experimental results, I found that this rank order is predicted almost exactly by counting the number of local subsymmetries in each pattern. The strips at the top (such as the one labeled **15**) have nine local symmetries, those near the bottom have five, and the others lie in between (see accompanying diagram, which enumerates the nine local symmetries in pattern **15** and the five symmetries in pattern **13**).

Figure 39 The local "subset" symmetries that occur in a system of 35 different strips

What is most significant is that the presence of these local "subset" symmetries in the pattern, and the number of them, appear to *cause* what is seen as coherence. This structure is obviously not a product of symmetry *breaking*. It is an example of multiple symmetry *production*. In innumerable cases, especially in organic development, local symmetries (limb-buds for example) are created, and this is one of the most important phenomena in organic development.

In order to understand this source of harmony, it is necessary to see the symmetries as part of a system of overlapping nested sets, in space, each of which may take on local symmetries or not, within its own local frame. One view of a harmony-seeking computation, in this context, is that it is a type of computation that injects as many overlapping local symmetries as possible, into a finite framework. Salingaros has shown that such compressed systems of local symmetries are present precisely in the acknowledged great buildings especially of ancient society.[47] In his computations, the top scorer for presence of local symmetries, with some additional refinements coming from others of the fifteen properties, was the Parthenon.

Example 13. Snow Crystals

Another rich source of information about the partial deficiencies of algorithmic computations in morphological matters lies in the attempts to simulate snow crystal formation. Snow crystal growth has been simulated with partial success by Diffusion Limited Aggregation (DLA) methods, and by cellular automata (CA) methods. So far, the results of these simulations pale when compared with the extraordinary variety seen in real snow crystals. Bentley photographed some 5,000 snow crystals during his lifetime, and his precise and exquisite photographs show us the kinds of structures which a successful computation must be able to create.[48] In particular, we see extraordinary LOCAL SYMMETRIES, DEEP INTERLOCK, POSITIVE SPACE, and GOOD SHAPE, in snow crystals, at levels that are common in works of art. These features do show up, but only weakly, in the results of the DLA or CA simulations so far published.

The currently prevailing theory of snow crystal growth says that the crystal grows outward, from a small hexagonal plate that is the starting point. The six-fold symmetry occurs in the six arms, so it is hypothesized, because the conditions of temperature and spatial constraint are essentially the same along each of the six radial axes – at a given moment in the history of its growth. Thus even though changing, the configuration is roughly the same on each arm as the growth moves steadily outward. Hence the high level of morphological similarity from arm to arm. This idea has been put forward with clarity by Libbrecht in the Department of Physics, at Cal Tech.[49]

But there are serious anomalies. If you examine the left-hand crystal illustrated below, you see that in addition to the six-fold symmetries that are indeed present, there are other symmetries. Most noticeable is an alternating pattern: three arms have one pattern, and the other three alternating with them, have another. This may be seen in the moth-like figures that occur on the three arms that are 120 degrees apart. The second illustration, a simple plate crystal, shows another fascinating aberration. Again, we have relatively uniform six-fold symmetry, centered on the six vertex axes as usual. However, if we look at the outer edges of the hexagon, along the three faces pointed to by the arrows, we see that they sides have a different symmetry: a shape that has two ogive S-curves (rather like a valance on the top of an 18th-century cupboard) just along the outer edge of the hexagon. This new symmetry is not only three-fold instead of six-fold, but it is based on three axes that are edge-centered, not vertex-centered. In another example published by Bentley, there is a configuration where a figure that appears within a conventional six sided plate, shows up only twice, 180 degrees apart, thus introducing an anomalous symmetry (see lower photograph).[50]

These non-six-fold phenomena do not seem to be explained by Libbrecht's assumption that the crystal's growth is all caused by uniform temperature variations on all six arms at the same time. Instead, there must be some mechanisms that generate a variety of symmetries, and it seems that it is the symmetries themselves which need to be the focus of the operation, not the growth.

Figure 40 and Figure 41 Two snowflakes with three-fold symmetries (axial and face-oriented) that appear within the six-fold scheme.

Figure 42   A two-fold symmetry, appearing(as if by chance) within the six-fold scheme.

These three examples of alternating three-fold symmetry and two-fold symmetry require some other explanation than simple growth outward under time-dependent spatially uniform temperature changes. Some larger W-E-transformation is entering into the computation. Clearly this highly ordered regularity cannot be explained by similarities merely caused by uniform temperature conditions from axis to axis. The three axes of symmetry now coming forward are not a subsystem of the original six growth axes, but three new ones that are interlaced among them. I believe (but have not yet shown how) the phenomenon can be explained by strongly invoking the LOCAL SYMMETRIES transformation, but not as a part of the simple kind of growth mechanism described by Libbrecht. Libbrecht's simulation may be seen at work on his website.[51] Long ago, further information was obtained by Ukichiro Nakaya, who observed and photographed thousands of snow crystals, and also experimented by growing them under controlled conditions.[52] In any case, it seems that only a model based on a nested, multi-level system of symmetries and axes of symmetry can ultimately provide an adequate explanation. Such a model may be a step towards a capability of modeling the general character of highly ordered structure at many levels simultaneously.

WHOLENESS-EXTENDING TRANSFORMATIONS AND COMPLEXITY THEORY

A Small Modification Needed in the Work of the Modern Masters and Complexity Theory

What I have said in the forgoing, can be simply summarized. Consider the modern masters: Benoit Mandelbrot, Brian Goodwin, Ian Stewart, Przemyslaw Prusinkiewicz, Deborah Gordon, Craig Reynolds. We must, and do, take off our hats to them. But if we examine the contributions made to complexity theory, we see that while making enormous strides, and placing our discipline on its first legs, it must also be said that each of them has fallen slightly short in one all-important respect. All of them, as far as I know, have tried to explain complex emergence as a product of coupled interactions among local events. Yet, when one examines in detail what is actually going on, and what is emerging, it turns out in every case that there is some aspect of the emerging whole that cannot be properly explained by this approach. The aberrations are small, but they are not small enough to be overlooked. Though they aspire to explain the whole that emerges through analytical means, in fact it is just that whole – the real whole in the world that makes us marvel in the first place – that is propelled by a second whole-driven process, which is both whole-driven and whole-oriented.

I reiterate: this whole-seeking or harmony-seeking process is not teleological, not goal-seeking. Instead, it comes about because of a new type of operation performed on the structure that exists, that brings to fruition a larger, unexpected, and unanticipated new structure of wholeness, in each individual case. The existence of such a computation, and its operation in virtually every creative process in nature and in art, perhaps embodies the real creativity of the universe at work.

I believe there is a realistic chance that this creative kind of emergence (different from the emergence currently under discussion), where the whole inspires the emergent structure, and gives it direction, can, at least in part, be encompassed by a computational formula, by a rational and attainable, though very new, mathematical formulation.

This hinges on the construction of a formalism in which nested systems of symmetries and centers are acted on by the transformations I have described, and where the future (the (t+1)-trajectory in phase space) is determined by action on the latent structures already present at time t, where these latent structures, too, are couched and described in terms of symmetries and centers.

USING HARMONY-SEEKING COMPUTATIONS IN BUILDING DESIGN

In natural systems we may expect, and in human-created urban systems we may decree, that when things are "okay", harmony-seeking computations are occurring.

Next I show a plan for a new housing project, in Massachusetts, where the working through of the plan, was done consciously, by following a sequence of harmony-seeking computations, in the fashion sketched in the paper.

Example 13. A Housing Project for 200 Apartments in Boston, Generated by Harmony-Seeking Computations

Figure 43 Perspective overview of a Boston project with some 200 condominums

Cycle 1. The process starts with a triangular brown-field site in Somerville, 5.5 acres in area, between a railroad line, a bike path, and an existing neighborhood → The first step in the computation then identifies latent centers in the site: the bike path, along the south side, Warwick avenue along the west, and Lowell Street in the south east corner.

Figure 44A Step 1

Cycle 2. latent centers in the site: the bike path, along the south side, Warwick avenue along the west, and Lowell Street in the south east corner. → The computation then identifies a connection and pedestrian precinct generates a curved line, more or less a median through the triangle of the site, but curving slightly and leading to stairs at the east end where there is a 20 foot rise to Lowell street.

Figure 44B Step 2

Cycle 3. Consideration of density suggests that the environment can become wholesome and enjoyable, only if entirely pedestrian, thus connecting this new area to the surround neighborhood and making it useful.

Figure 44C Step 3

Cycle 4. The articulated pedestrian spine now has, as its most obvious latent centers, the areas on either side of it. To place 200 dwellings in this limited area, and to do it in such a way that people have pleasant gardens, for their own use, and for the neighbors who may like to enjoy these gardens, leads to a form of organization where houses are laid in long thin strips around useful open spaces. The gardens should, typically, have a diameter on the order of some 100 feet; the buildings in strips should have a depth of some 25 feet; the building height would then be 2 and 3 stories.

Figure 44D Step 4

Cycle 5. The abstract and schematized grid-like array, is characterized by two parameters, courtyard diameter, and building thickness. However, its regularity is not essential or even good: indeed it needs to be adapted to the site boundaries in such a way as to generate coherent courtyards and pedestrian space, while leaving building volumes simple. This transformation requires the use of ROUGHNESS, LOCAL SYMMETRIES, DEEP INTERLOCK, NOT SEPARATENESS, GOOD SHAPE and INNER CALM. Application of these transformations in concert achieves the necessary computation at this stage.

Figure 44E Step 5

Cycle 6. More careful adaptation. The schematic arrangement of the previous cycle, now gives way to a series of shapes which pay more detailed attention to each individual garden, as a shape in itself, using GOOD SHAPE, LOCAL SYMMETRIES, POSITIVE SPACE, THE VOID, and ALTERNATING REPETITION, so that the whole is coherent, and feels like one thing.

Figure 44F Step 6

In addition, the boundary where this neighborhood meets and abuts other neighborhoods is modified by BOUNDARIES, DEEP INTERLOCK, and ALTERNATING REPETITION, so that it becomes a thickened semi permeable membrane, capable of allowing people who want to talk a stroll, to pass in and out comfortably, yet also maintaining a certain privacy for the interior of the neighborhood.

Figure 45 Final plan, after the six steps I have described, and a further twenty, comparable steps (not illustrated here)

In the plan above we see the beginning and the framework for a further harmony-seeking process. The plan shows 200 households on a five acre site, each house potentially unique, and where the pattern of gardens, walkways and road access have been laid down so as to protect the harmony of the adjacent neighborhood, and project the immediate environment for 500 people.

The Uniqueness of Each Region in the Generated Structure

One aspect of the generated structure for Somerville is highly significant. If we examine the structure generated by the harmony-seeking computations, we see that each part, though similar in broad structural character to others, is *unique*. That comes about because *the application of the computation to* *even slightly differing contexts inevitably produces morphologically different results. Furthermore, even a slight difference of contexts, will, after a few cycles of wholeness-extending transformations, will magnify the difference, and lead to very substantially different configurations. This is extremely important, and is the origin of uniqueness in the world. A world with genuine, and profound uniqueness at every spot, will arise whenever harmony-seeking computations govern the unfolding.*

For example, at the largest level of scale, each of the dozen or so courtyard gardens has a different shape and configuration. That occurs not because differences are arbitrarily inserted to be cute, but because the application of the principal transformations (POSITIVE SPACE, LOCAL SYMMETRIES, ECHOES and GRADIENTS, ALTERNATING REPETITION, CONTRAST), generates a different configuration for each one according to its starting point – that is, according to its context.

Figure 46 Different gardens in the project, each with its unique character and atmosphere

Figure 47 Detailed view of the houses and apartments, showing the detailed adaptation and variety which arises from the use of harmony-seeking computations

Still more exciting is that the same quality of uniqueness and subtle differentiation continues to smaller and smaller scales. The detailed configuration of terraces, entrances, paths, lawns, stairs and archways, produces unique results in each part of the larger whole, and in each part of the individual gardens. This is not from a shallow desire to make each thing different for its own sake (sometimes the driving force behind the more commercial postmodern housing developments). It occurs because the effect of harmony seeking computations, on only slightly different starting conditions, is to generate entirely new and different configurations, but all members of a fairly simple family.

CONCLUSIONS

A Single Conclusion From These Studies

There is a single all-important conclusion to be drawn from all these studies. Atomistic, bottom-up computations cannot adequately describe what is really happening in the world, and, furthermore, do *not* describe those especially important processes that *heal* the world, that bring order into configurations in the land, in nature, in buildings deeply adapted to the land, and also deeply adapted in their internal structure.

The issue is recursive. It is not only large-scale configurations that need to benefit from this insight. Every configuration, at every level, is working to help the coherence of some larger configuration in which it is embedded. This runs up and down the ladder of scales, and must do so in any living world. A successful computation will emulate this upwards- and downwards-reaching process. Purely bottom-up forms of calculation, not invoking this principle, will always remain too sterile to be real or profound.

What is the Underlying Process Involved in These Harmony-Seeking Computations

I am not only proposing that we consider these many real world systems as computational processes. I am saying that we need to find out how they work, and how, in particular, they work to allow the unfolding of structure, under the impact of wholeness. This requires a form of representation that is new. It has been sketched, in broad brushstrokes, in Books 1 and 2 of *The Nature Of Order*. We now need to work at finding ways of describing this kind of unfolding in more well-defined mathematical and computational terms.

That will be a very long job. My colleagues and I have, in the last few years, gained intuition and insight into the nature of the harmony-seeking computations. But we are far from understanding them in detail. That enormous task must now be undertaken, hopefully by many dedicated scientists together.

I must repeat that the phenomena I am talking about cannot simply be grouped under what is loosely called emergence. The emergence of wholes in the world does not come about by autonomous processes that happen to aggregate themselves to form wholes. Rather they are the wholeness-extending processes I have described.

Wholeness-Extending Transformations

In all these real-world examples, there is a common phenomenon. We may describe it by saying that the steps of the computation are W-E (wholeness-extending)-transformations which follow this scheme. Each W-E-transformation operates on one wholeness to produce another wholeness. It does it in such a way as to preserve or embellish or enhance the global structure of the first wholeness.

$$W_1 \xrightarrow{WE_1} W_2 \xrightarrow{WE_2} W_3 \xrightarrow{WE_3} W_4 \xrightarrow{WE_4}$$

The mathematical description of an W-E-transformation is not yet fully known. However, there is abundant evidence to show that the concept of being W-E is well-defined and objective, in the sense that different observers largely agree among that different possible transformations of a given whole, which ones are more W-E and which are less so.

Models of the Wholeness in a Given Configuration?

To establish the character of W-E-transformations, we begin with five postulates about the structure of wholeness:

Postulate A1. In any configuration we see certain salient wholes, or centers. Each of these wholes is an identified, spatially contiguous subset of the configuration that corresponds to something we see, or experience, as an "entity".

Postulate A2. The sub-configurations may be spatially nested, or overlapping, or disjoint.

Postulate A3. Each sub-configuration has a measure associated with it, the degree of coherence, or saliency within the larger whole.

Postulate A4. Certain *latent* centers may be very low saliency, almost invisible, but are nevertheless coherent configurations in their own right, created by others in the configuration.

Postulate A5. The wholeness is defined as the system of configurations, each one specifying its coherence, and each connected with other configurations which are part of it, or of which it is a part.

And three postulates about the definition of W-E:

Postulate B1. A transformation is considered to be W-E if it elaborates existing centers or latent centers, and does not introduce new centers that violate or 'cut across' existing centers.

Postulate B2. The best latent center to work on is the latent center whose improvement or repair, will (probably) do the most to increase the coherence of the whole configuration.

Postulate B3. A successful W-E transformation must always have morphological impact on the structure of some larger whole.

A Few Randomly Chosen Examples of Harmony-Seeking Computations

Figure 48 → THICK BOUNDARIES, THE VOID → LEVELS OF SCALE, ALTERNATING REPETITION, LOCAL SYMMETRIES

Figure 49 → STRONG CENTERS, LEVELS OF SCALE → LOCAL SYMMETRIES

Figure 50 → THICK BOUNDARIES, THE VOID → GRADIENTS, LEVELS OF SCALE, ALTERNATING REPETITION, ECHOES

Figure 51 →POSITIVE SPACE → GRADIENTS, LEVELS OF SCALE → POSITIVE SPACE

Figure 52 →STRONG CENTER → LEVELS OF SCALE → DEEP INTERLOCK, LOCAL SYMMETRIES

Figure 53 →GRADIENTS → SIMPLICITY → POSITIVE SPACE → LOCAL SYMMETRIES → CONTRAST

The Paradigm Being Followed in Each W-E-Transformation

Each of these examples is relatively simple: the same paradigmatic cycle is followed each time.

- Frame 1: a configuration
- Frame 2: identifies the locus and extent of some latent center in that configuration – one which, if consolidated, would help the whole to become more coherent.
- Frame 3: establishes smaller centers to embody and solidify that latent center.

The steps taken in going from frame 1 to frame 2, and the steps taken in going from frame 2 to frame 3, are always some conglomerate of the 15 fundamental W-E-transformations.

We may also see how symmetry elaboration is happening at each step. At each step, the space is being differentiated, holistically, by an injection of local symmetries. It is possible, as I have said earlier, to view this injection of local symmetries as symmetry breaking. The symmetries that appear, in most cases, appear because other symmetries, often an infinity of other symmetries, are struck out, leaving the coherent form we see. Because of this symmetry breaking aspect, the injecting of local symmetries is structure-preserving, not structure-destroying.

This is an extremely significant fact. However, it slightly obscures the core of what is going on to think of it, exclusively, in this way. Operationally, it is more illuminating to focus on the fact that local symmetries of various kinds are being *created*. But the kinds of local symmetry being created, when created *well*, do always enhance the underlying structure of what was there before.

Operationally, and emotionally, such a procedure is creating structure in a new way. It may be done by an artist, with an intuitive grasp of the underlying latent structure, at each step in the unfolding of the whole. Or it may be done by an engineer. Socially speaking, this is a new kind of process, a new function for an artist, and a new challenge to engineers, architects and planners of all kinds. This new kind of work demands enormous concentration and attention.

The intuitive act is nevertheless a computation, and we may be able to pin down what *kind* of computation it is. Then, if we can succeed in making a harmony-seeking computation, even perhaps one day helped by a computer working in a new way to achieve similar holistic results, *that will be because the thing we recognize intuitively as coherent or whole, is, mathematically, a particular recursively generated structure of symmetries and centers which have the 15 properties in them*. It is this underlying structure that allows the human mind, and natural processes, both, to follow this path and to seek wholeness in the way they do. Most important, we may become conscious about this process, and consciously use this kind of computation to improve the coherence and harmony of our world.

Fact and Value

The architectural examples and the example of the cosmological structure of voids and filaments bring the potentially extraordinary nature of harmony-seeking computations into sharp focus.

In the parlance of 20th century thought, designing a building is a matter of personal taste; it has essentially nothing to do with fact. On the other hand, the dynamics that lie behind the distribution of matter in the universe is undoubtedly a matter of *fact*, although the facts may be still poorly understood and much debated.

No matter how subtle my perception of a building design and its harmony might be, the idea that I treat its structure as a matter of fact, may seem absurd to present day scientists, if they follow the canon of 20th century thought. It is an article of contemporary faith, that the goodness of a building is a matter of *taste*, not fact. The harmony of the building cannot be (according to 20th century thought) a matter of fact in any sense. The mental protocols of 20th century thinking have forbidden it.

Here we come to the profound change that has been lying in wait for 21st century science. It is a matter of historical record that scientists of earlier eras – indeed, many of the great scientists of earlier eras – had no difficulty whatever thinking of the great harmony that existed in the world. Pythagoras's phrase "the harmony of the spheres" was not an idle one. Newton, as a matter of record, considered the progress of the universe, in the large and in the small, to be entangled, inevitably, with a movement towards harmony, and with the greater harmony of the world as a necessary underpinning for the discoveries of science.[53] Leibniz, Kepler, and others thought the same.

Considering a building as a matter of taste strongly limits our ability to understand its harmony. Considering the evolution of the universe as a mechanical product that can best be modeled through value-neutral means, also strongly limits our ability to understand the ensuing harmony, and in large-scale cases like this one creates a real possibility that we shall ultimately fail to understand the physics.

These two statements may seem hard to swallow. But the possibility of harmony-seeking computations ventures, precisely, into this forbidden domain. It is challenging, undoubtedly, to succeed in defining and expanding this kind of concept sufficiently well, so that it can become an effective part of the way we think about the world. But it may play a considerable part in reopening a door that has been closed for too long.

A New Science of Harmony-Seeking Computation: When and Where?

A new science of harmony-seeking computation and W-E-transformations can make amenable to computation phenomena that are, for the moment, altogether beyond the reach of currently available computational methods. It can help to open doors to the global quality of harmony, figural goodness, ecological health and structural coherence as a computable feature of configurations.

One first practical item on the agenda is to provide well-defined and precise versions of the 15 transformations in *The Nature Of Order*. Though easy to state, this is a remarkably difficult task, for three reasons. Firstly, the 15 properties, though defined with some level of precision, remain somewhat elusive. Defining computational operations that can induce these properties in arbitrary configurations is a challenging task. Secondly, it is difficult to define them as transformations, since this presupposes a language of configurations that is amenable to the transformations. Thirdly, some of the transformations are easier than others to define operationally in sufficiently concrete terms. For example, LOCAL SYMMETRIES, THICK BOUNDARIES and LEVELS OF SCALE are relatively easy. POSITIVE SPACE and ECHOES are harder. SIMPLICITY AND INNER CALM and NOT SEPARATENESS are among the most difficult.

Despite these difficulties, I am fairly sure that the task of dealing with all 15 transformations can be accomplished by a small team in the next five years, and that it should then open the door to a full fledged, though elementary, version of a first-draft science of harmony-seeking computations. First steps have already been taken.[54]

I hope the idea of harmony-seeking computation may then sit alongside other methods as a new tool in an armory of well-founded computational techniques to be used when appropriate. It is likely to be appropriate whenever a computational task is defined more by issues of adaptation, health, wholeness, and wellness, with reference to the position some system in some still larger whole, or perhaps even by a desire for beauty, or life, or elegance.

All these might one day play a key role in very general kinds of computation. Science, architecture, biology, ecology, physics, cosmology – and computation – may all be the better for it.

NOTES

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1 Christopher Alexander, *The Nature of Order, Four Volumes*, Center for Environmental Structure Publishing, Berkeley, California, 2002-2005. Abbreviated in subsequent notes as *NOO*.

2 Martin Golubitsky and Ian Stewart, *The Symmetry Perspective*, Basel, Birkhauser, 2001.

3 Christopher Alexander and A.W.F. Huggins, “On Changing The Way People See”, *Perceptual And Motor Skills*, Vol. 19, July, 1964, pp. 235-253.

4 Ibid, 1964.

5 Alexander, op.cit, 2002-2005.

6 Coherence of a center, and strength of a center are two ways of referring to the same quality that centers have in greater or lesser degree. The degree of this quality that is present in a

center, indeed in any whole, can be measured by empirical methods described extensively in *NOO* Book 1, chapters

[7] In *NOO*, the property is called simply BOUNDARIES. I have since begun using the phrase THICK BOUNDARIES, since it more clearly distinguishes the boundaries which work most effectively to create coherence, and also eliminates confusion with the common mathematical use of the word boundary as a dimensionless interface.

[8] *The Nature of Order*, *Book 1 The Phenomenon of Life,* pages 143-296.

[9] *The Nature of Order*, *Book 2 The Process of Creating Life,* pages 65-84.

[10] Studies of co-evolution and ecological evolution have certain moved in this direction. But they have rarely concentrated on the geometry of the evolving system as the key variable of interest.

[11] *NOO*, Book 2 (especially chapter 2), and Books 3 and 4 throughout.

[12] Susan Stepney was kind enough to raise this question: What if the existing structure is ugly? Can it be made beautiful by this process (a la sand in an oyster) or should one not start from there? The answer is, in varying degrees, both. In nature since the structure generally grows from almost pure homogeneity, the beauty is there from the beginning, and continues there as the system evolves. Among humanly made urban landscapes, where ugliness is common, it is tempting to teardown and start again: however, I usually resist this temptation, and leave much of the ugly part, pruning out only the worst. When wholeness-extending transformations are applied to this pruned ugly place, it soon brings the system back to order, and leads to more wholesome results.

[13] See the discussion of four Matisse self portraits, *The Phenomenon of Life*, page 97.

[14] Max Wertheimer, Wolfgang Koehler, Kurt Koffka.. etc.

[15] Christopher Alexander and A.W.F. Huggins, “On Changing The Way People See”, *Perceptual And Motor Skills*, Vol. 19, July, 1964, pp. 235-253.

[16] Hochberg J. & McAlister E. (1953) A Quantitative Approach to Figural "Goodness." *Journal of Experimental Psychology* 46, 361-364.

[17] Christopher Alexander and Susan Carey, "Subsymmetries", *Perception And Psychophysics*, Vol. 4 (2), February, 1968, pp. 73-77.

[18] Photograph of hayricks in Romania by Radu B. Chindris

[19] Gerry Webster and Brian Goodwin, *Form and Transformation*, Cambridge University Press, 1998, pages 193-216.

[20] *Form and Transformation*, op cit. pages 209-230.

[21] The Emoto Building, in Komagome, Tokyo, illustrated in *The Nature of Order*, Book 3, pages 166-73.

[22] ***Op.Cit.*** The fifty steps are summarized on pages 167-71.

[23] See *The Nature of Order*, *Book 1*, chapter 5, and *Book 2*, chapter 2.

[24] Historical data and additional analysis of the evolution of St. Mark's Square are provided in Books 2 and 3 of *The Nature of Order:* Book 2, *The Process of Creating Life*, pages 251-55, and Book 3, *A Vision of a Living World*, pages 5-7.

[25] Steve Shechtman et al., *Astrophysical Journal*, **470**, 1996, page 172.

[26] Simulation and photograph by George Lake and Tom Quinn, published in Greg Bothun, *Modern Cosmological Observations and Problems*, Taylor and Francis, 1998, figure 3.1.

[27] Nature of Order, Book 2, pp.51-84, especially page 59.

[28] Christopher Alexander, *The Process of Creating Life*, Berkeley, California, Center for Environmental Structure Publishing, 2003, pp. 18-84.

[29] Christopher Alexander, *The Phenomenon of Life*, Berkeley, California, Center for Environmental Structure Publishing, 2002, pp. 143-296.

[30] Cathryn J Polinsky, *Flight Simulation of Flocking Geese Using Particle Set Animation,* Swarthmore College, May 1999

[31] Deborah Gordon, *Ants at Work: How an Insect Society is Organized,* New York, Free Press, 1999.

[32] slime mold references.

[33] All reported in Stephen Johnson, *Emergence*, New York 2001.

[34] Debra Niehoff, *The Language of Life*, Joseph Henry Press, Washington DC, 2005

[35] Craig Reynolds, Flocks, Herds, and Schools: A Distributed Behavioral Model, the SIGGRAPH '87 boids paper.

[36] Cathryn J Polinsky, *Flight Simulation of Flocking Geese Using Particle Set Animation,* Swarthmore College, May 1999

[37] Mitchel Resnick et al, *StarLogo 2.1*, Media Lab, Massachusetts Institute of Technology, 2004.

[38] Lene K. Hjertager, Bjørn H. Hjertager, Niels G. Deen, Tron Solberg, Measurement of turbulent mixing in a confined wake flow using combined PIV and PLIF, Submitted to the *Can. J. of Chem. Eng*., 2003. A sophisticated algorithm would have to model the aerodynamics to identify the sweet spot. A much simpler (and more easily constructed) algorithm will merely need to identify the cone of angles in the wake, where the sweet spot is like to appear, and give positioning in that cone a high probability.

[39] Michael Cross, "Pattern Formation in non equilibrium systems", course notes from Cal Tech Physics 161b, 2000. See http://www.cmp.caltech.edu/~mcc/Patterns/index.html

[40] Radomir Mech and Przemyslaw Prusinkiewicz "Visual Models of Plants Interacting with Their Environment" Department of Computer Science, University of Calgary, Calgary, Alberta, Canada, also Proceedings of SIGGRAPH 96 (New Orleans, Louisiana, August 4–9, 1996). In Computer Graphics Proceedings, Annual Conference Series, 1996, ACM SIGGRAPH, pp. 397–410.

[41] Simulation of a tree by H. Honda, Description Of The Form Of Trees By The Parameters Of The Treelike Body: Effects Of The Branching Angle And The Branch Length On The Shape Of The Tree Like Body, *Journal of Theoretical Biology*, 31: 331-338, 1971

[42] Ian Stewart and Martin Golubitsky, *Fearful Symmetry*, New York and London, 1988.

[43] Martin Golubitsky and Ian Stewart, *The Symmetry Perspective*, Basel, Birkhauser, 2001.

[44] Reference to definition of wholeness

[45] Many physical examples of the presence, and density, of local symmetries in a structure is described in *NOO*, Book 1, pages 186-234.

[46] Christopher Alexander and Susan Carey, "Subsymmetries", *Perception And Psychophysics*, Vol. 4 (2), February, 1968, pp. 73-77.

[47] Salingaros, Nikos A. (1997). "Life and Complexity in Architecture From a Thermodynamic Analogy" in *Physics Essays,* vol. **10**, pp. 165-173

[48] Wilson A. Bentley, *Snow Crystals*, containing more than 2400 snow crystal images, was published by McGraw-Hill, 1931, but has long been out of print. A soft cover copy, identical in all respects, can be obtained today from Dover Publications, Inc.

[49] Kenneth G. Libbrecht, and Patricia Rasmussen, *The Snowflake,* Colin Baxter, 2004.

[50]

[51] For some simulation processes, see the website *SnowCrystals.com*, created by Kenneth G. Libbrecht, in the Department of Physics, California Institute of Technology.

[52] Ukichiro Nakaya, *Snow Crystals: Natural and Artificial*, Harvard University Press, 1954.

[53] Michael White, *Isaac Newton: The Last Sorcerer*, London, Fourth Estate, 1997.

[54] See for example, Salingaros, Nikos A. (1997). "Life and Complexity in Architecture From a Thermodynamic Analogy" in *Physics Essays,* vol. **10**, pp. 165-173; Christopher Alexander and Stuart Cowan, "*The Field of Wholeness:A Mathematical Model*", in progress, 2006; Cees der Groot, *First Steps In a SiteLayout Program written In Squeak*, 2004; Salingaros, Nikos, A Scientific Basis for Creating Architectural Forms, *Journal of Architectural and Planning Research* **15** (1998), pages 283-293.

IJUC illustrations

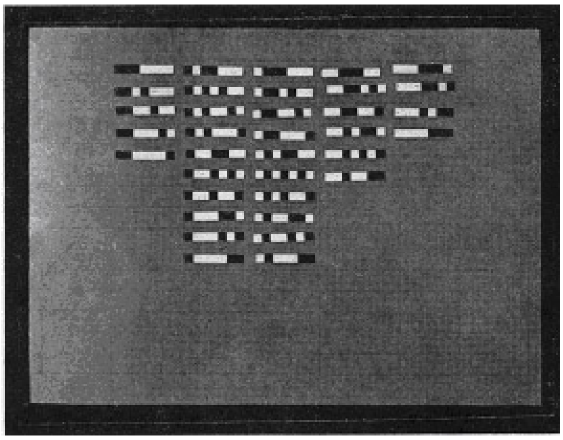


Figure 1   Sequential-digital: Reading the strips left to right

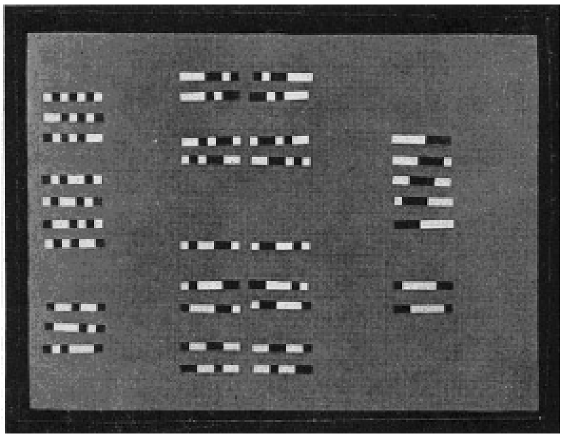


Figure 2   Figural-holistic: Seeing the strips as patterns

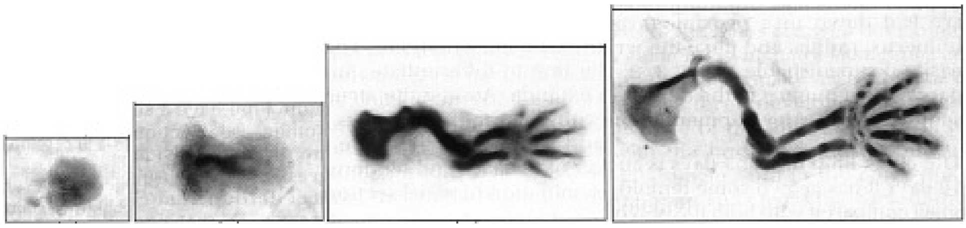


Figure 3   Step by step development of a mouse foot

Figures 4A, 4B and 4C   A delightful seat around a tree, being woven in basket form and then finished with turf

Figure 4D The finished seat

Figure 5 Hay ricks in Romania

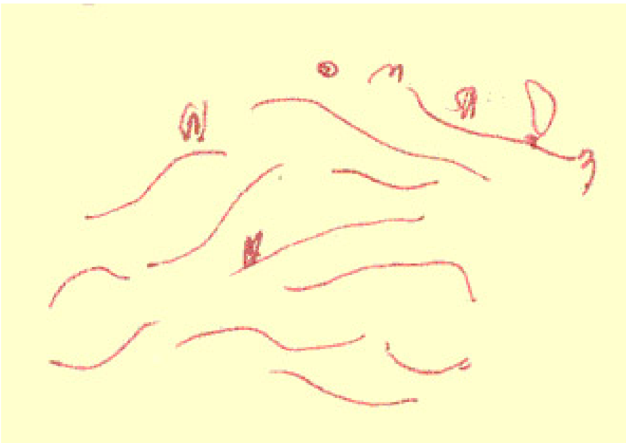


Figure 6   Impressionistic depiction of the wholeness structure in the land

Figure 7   Wind turbines on the flat north coast of Denmark

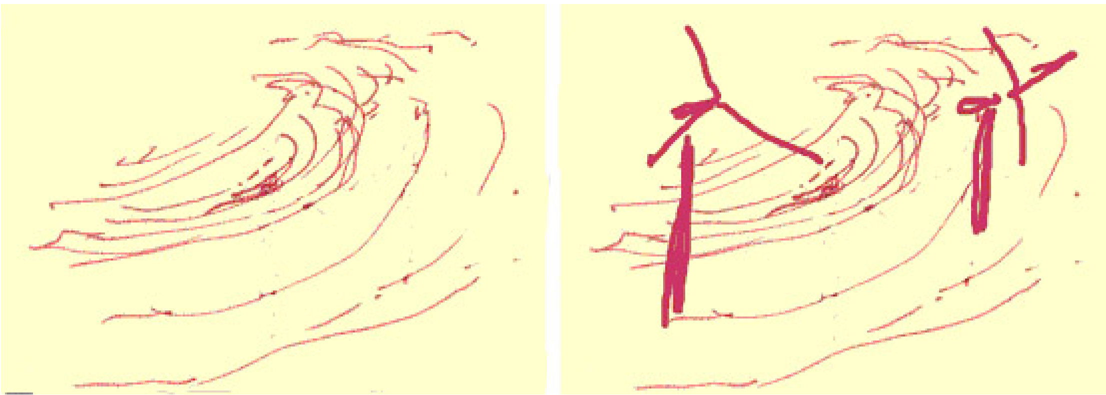


Figure 8A   Diagram of the wholeness before construction of the turbines
Figure 8B   Diagram of the wholeness after construction of the turbines

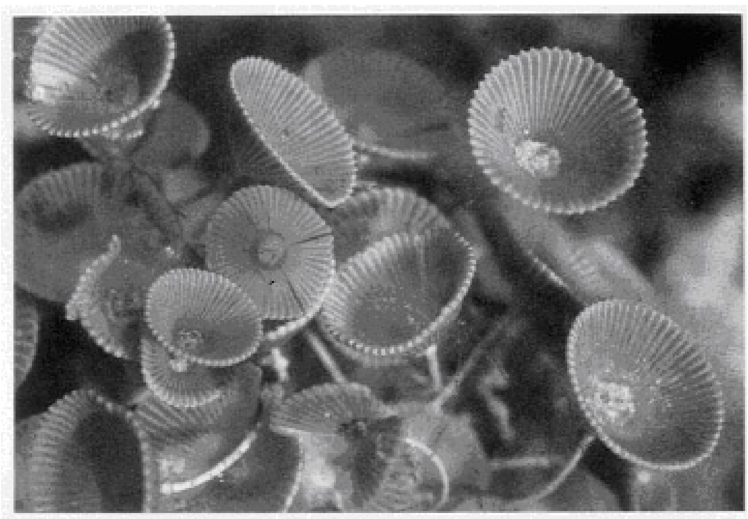


Figure 9   Whorls forming at the tips of developing Acetabularia

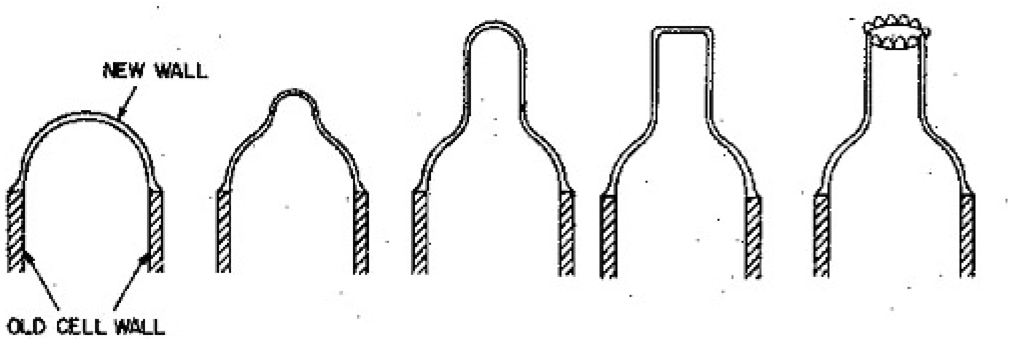


Figure 10   Goodwin's five diagram sequence

Figures 11A and 11B   One of the Wittenham clumps near Oxford and A diagram of the structure created on the mound by the prehistoric embankment

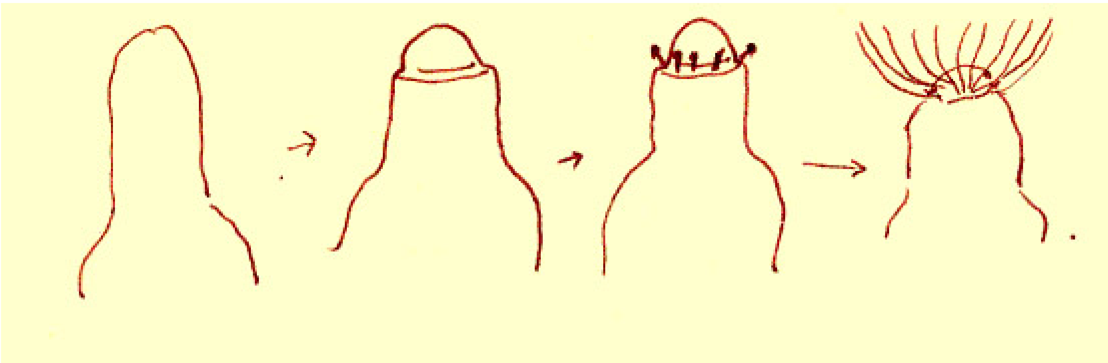


Figure 12 My analysis of the developmental sequence, based on W-E-transformations

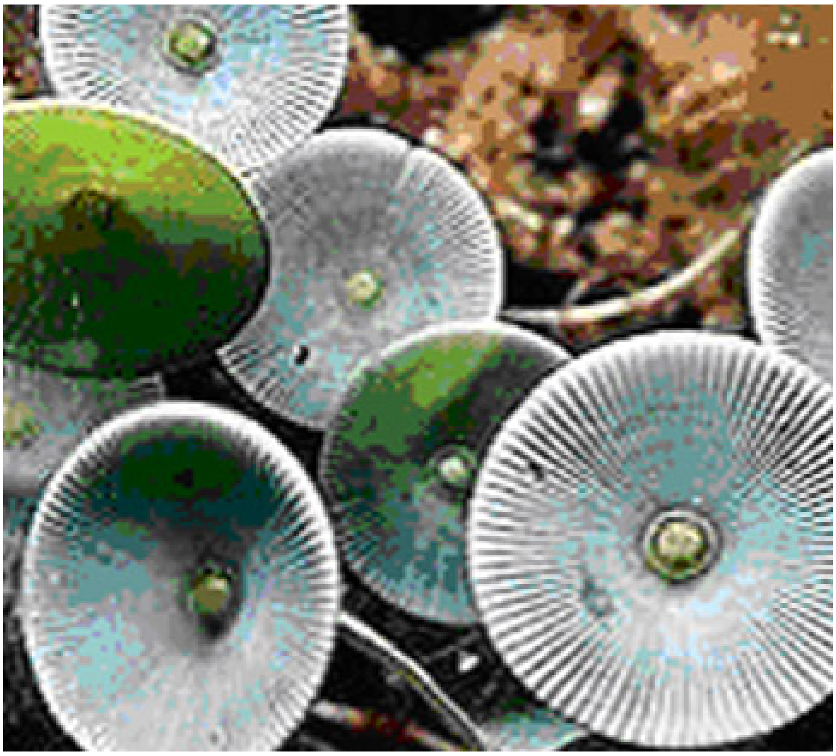


Figure 13 Enlarged photo of Acetabularia showing hillocks, not flats, at the center of the whorls

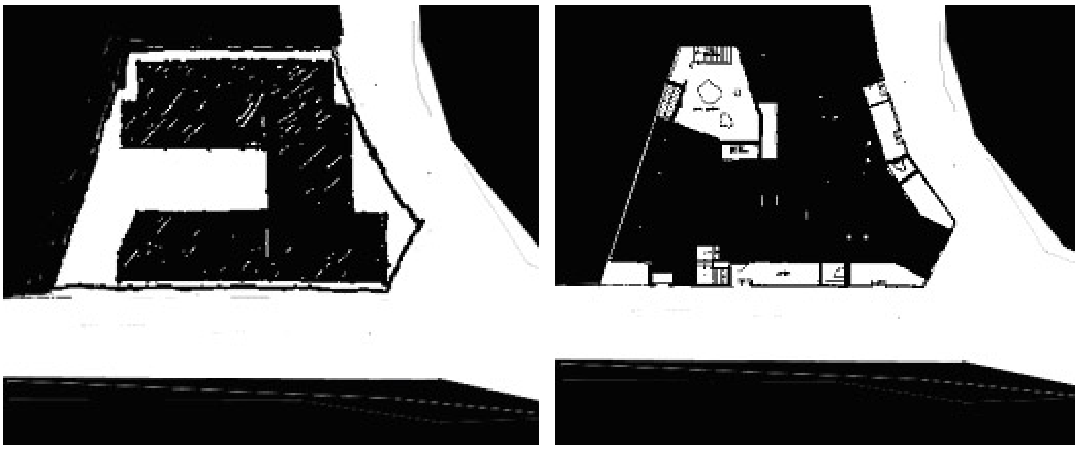


Figure 14A and 14B Two possible plans for the five story Emoto apartment building in Tokyo. South is at 10.30 pm.

Figure 15   The completed Emoto apartment building

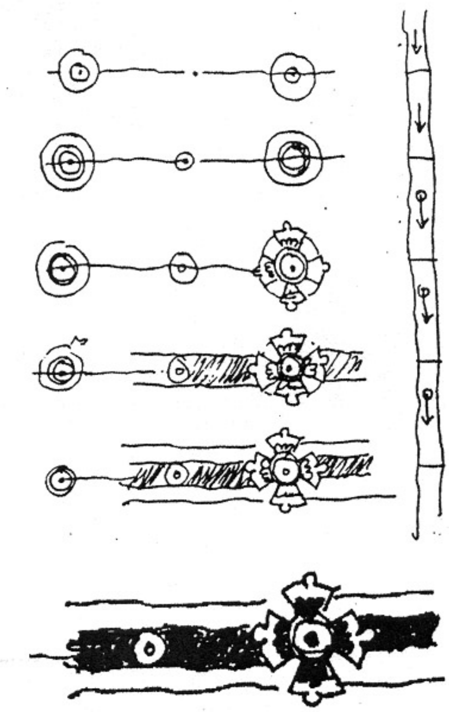


Figure 16   Six W-E-transformations leading to the creation of a new ornament

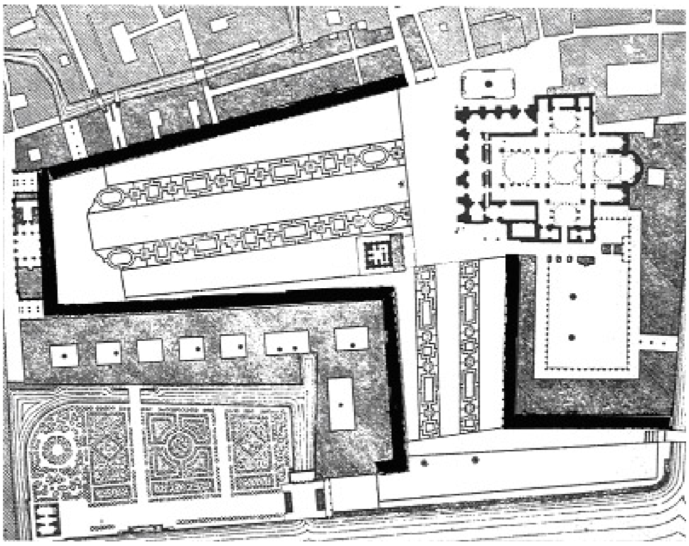
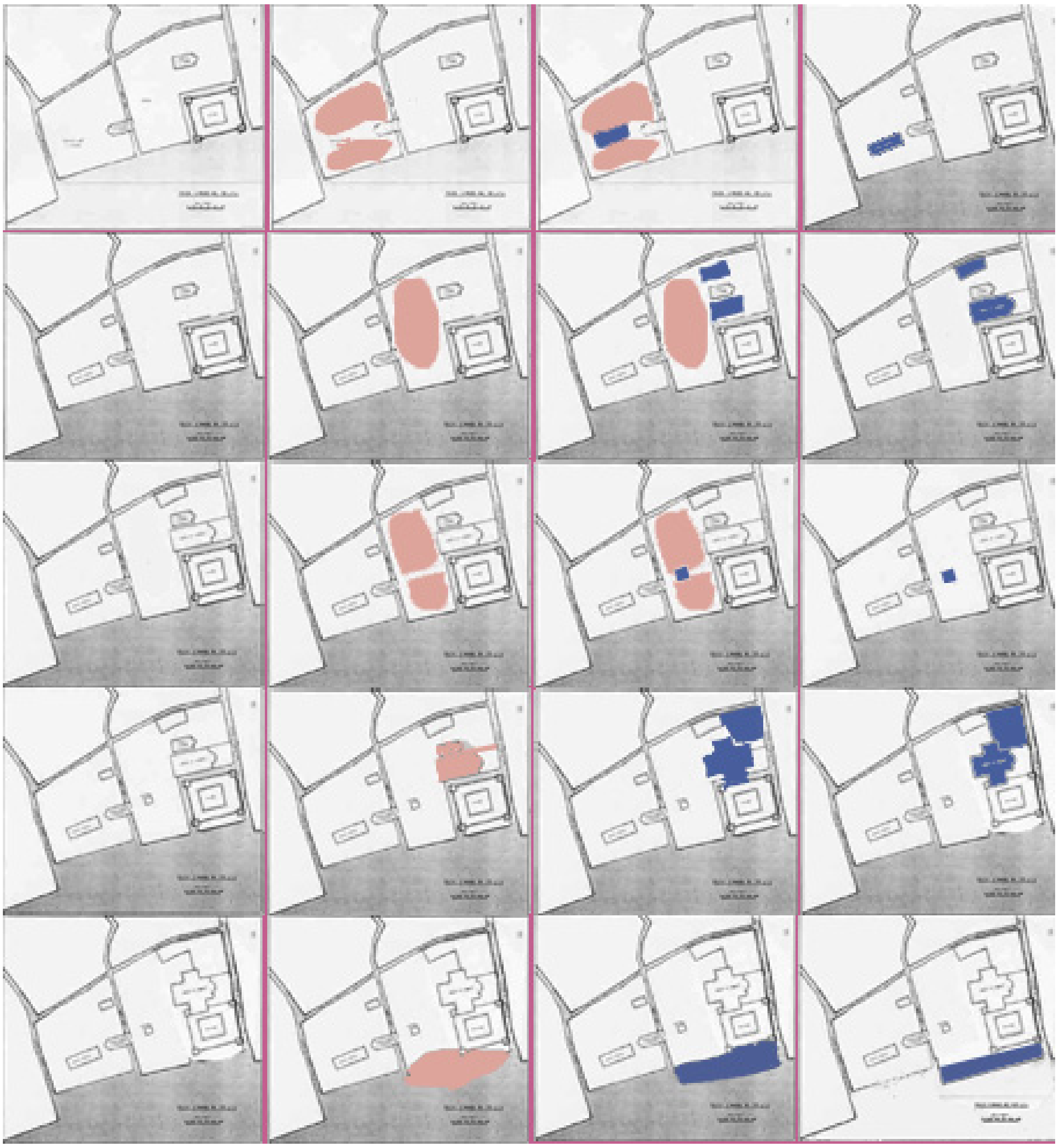


Figure 17   Plan of St Mark's Square



Figures 18A,B,C,D,E,F,G,H,I,J Ten cycles of W-E-transformations in the history of 1000 year development of St Mark's Square

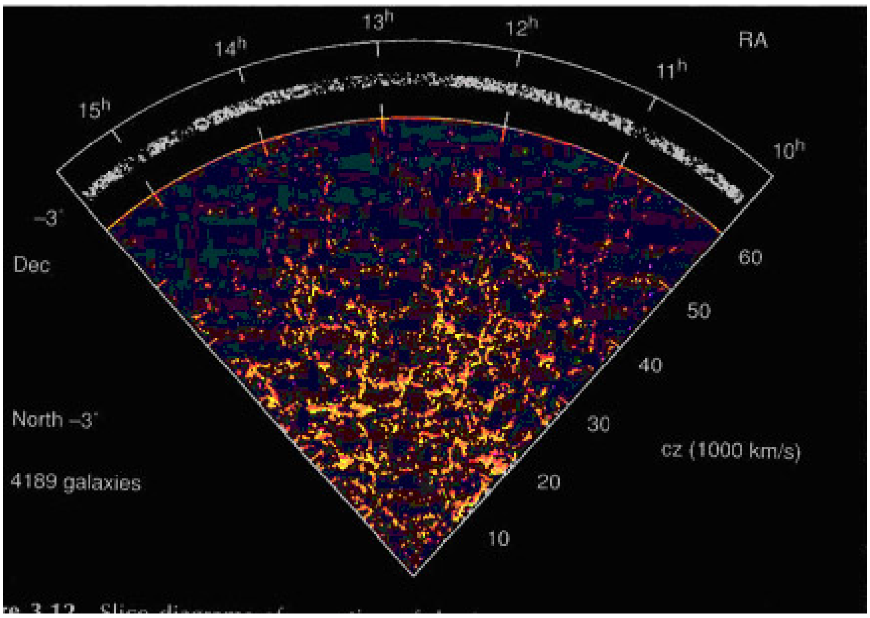


Figure 19 Shechtman et al, the Las Campanas Redshift Survey,1996, showing voids and filaments

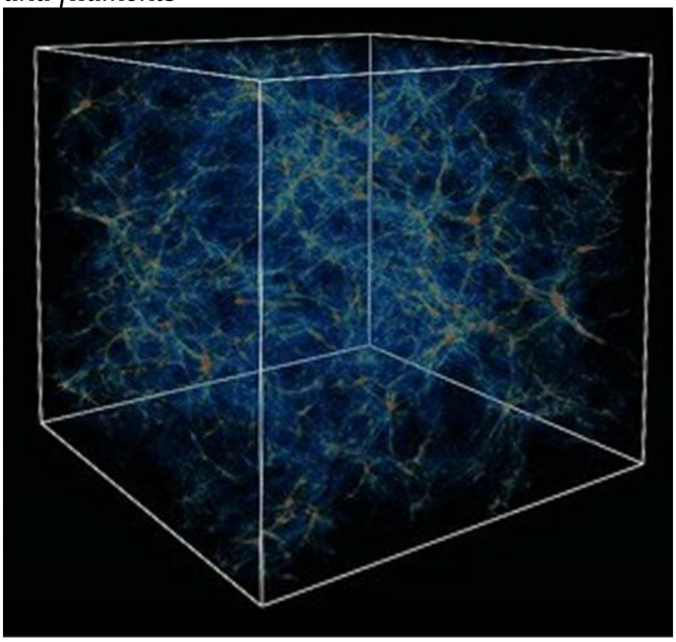


Figure 20 Artist's sketch of the 3D structure, showing roughly how the ring-like filaments lie in space, on the surface of the voids

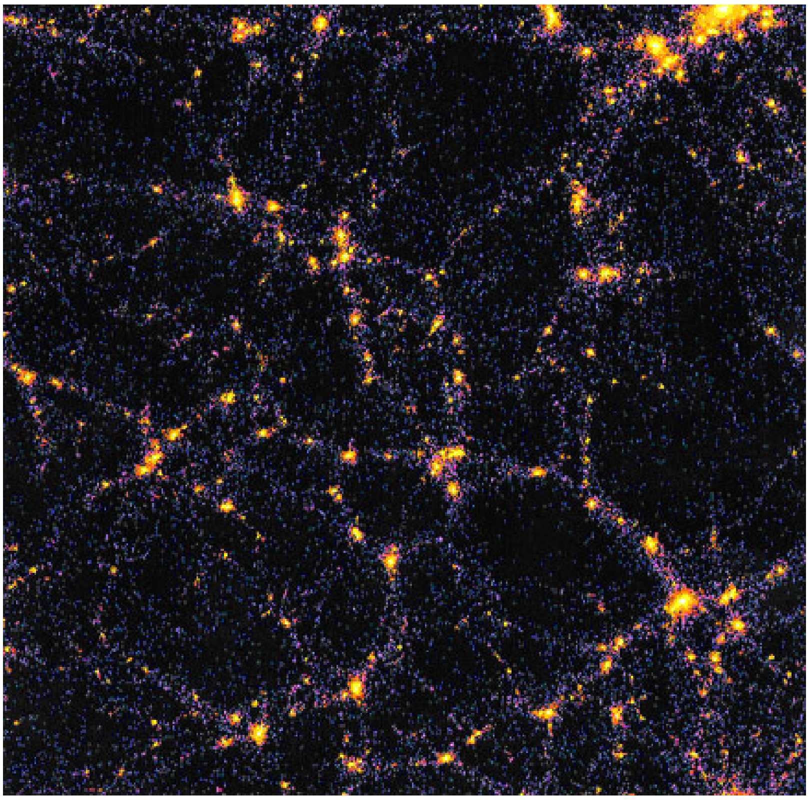


Figure 21 The Lake-Quinn simulation

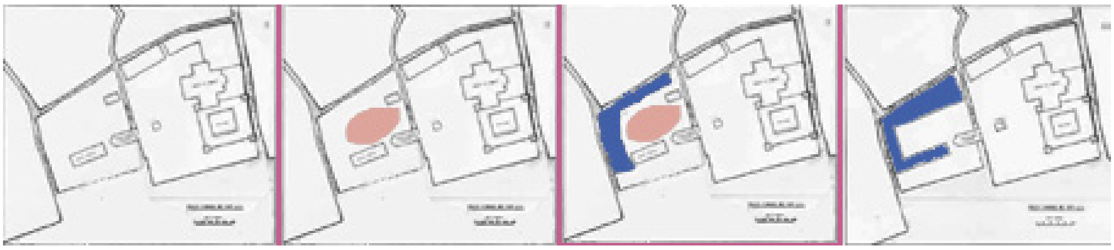


Figure 22 A second look at one of the W-E-cycles in St Mark's square

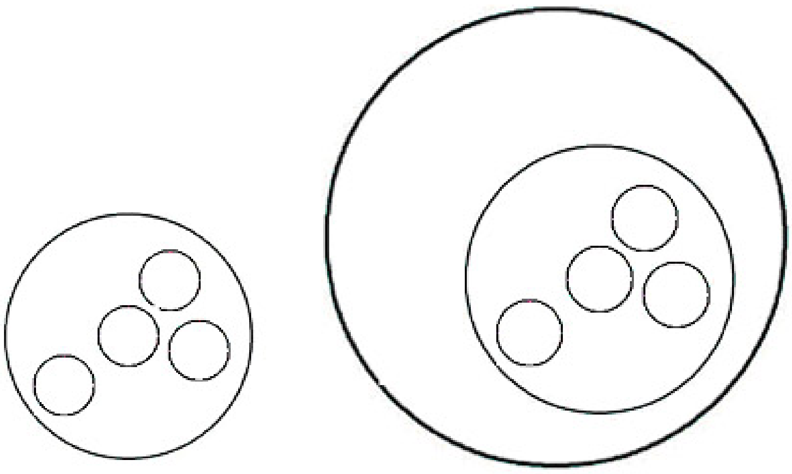


Figure 23 Emergence, a two level relationship
Figure 24 Harmony, a three level relationship

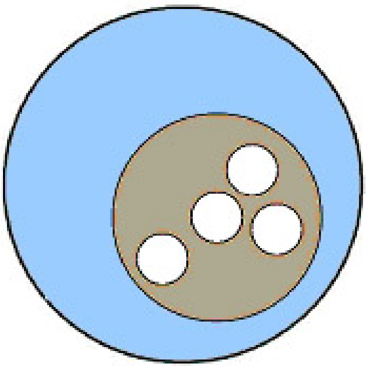


Figure 25   Key condition for the healing to take place in a W-E-transformation

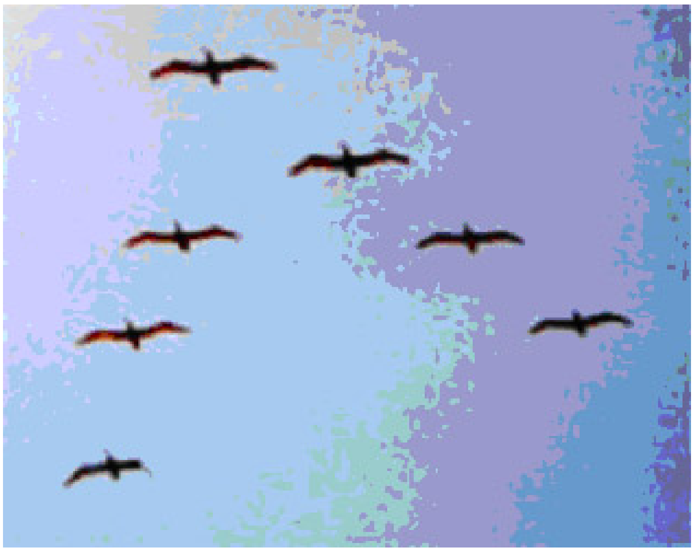


Figure 26   The V-formation of Canada Geese

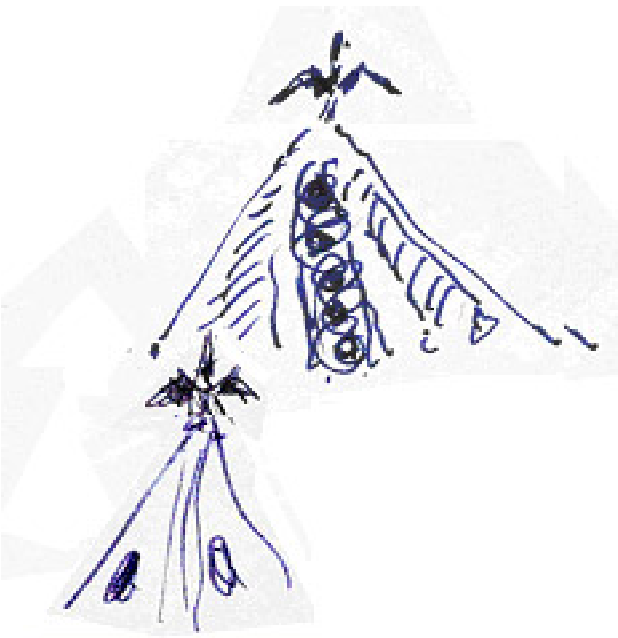


Figure 27   Each bird flies in the outer wake of the bird ahead of it, since that is the spot which takes the least energy –just like riding a bike behind a bus, to catch a ride from the negative pressure in the airflow.

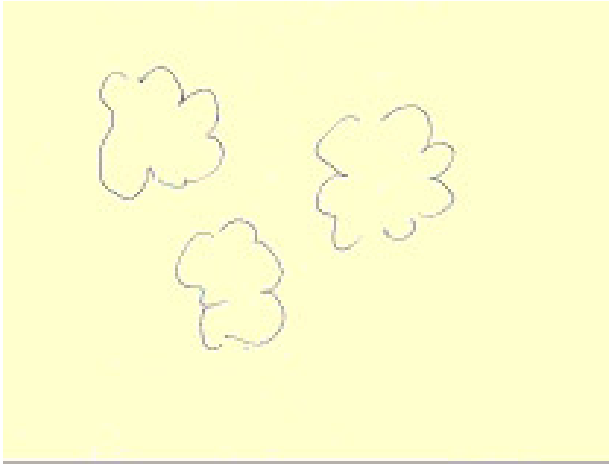
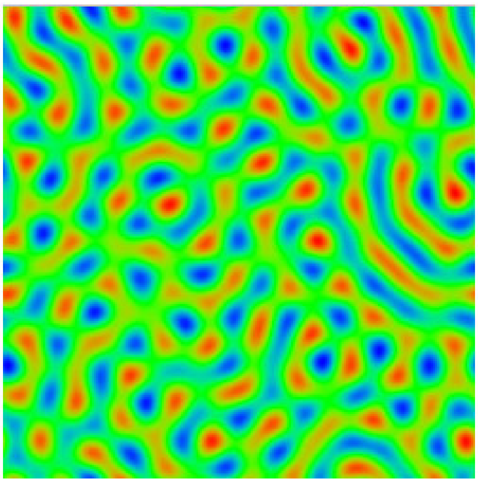



Figure 28   Cotton wool clouds: the clouds are objects and the sky is background
Figure 29   The real thing, a very different structure

Figure 30   Clouds and the sky between them, as they appear in nature

Figure 31   The blue systems are clearly visible here as having their own shape and character



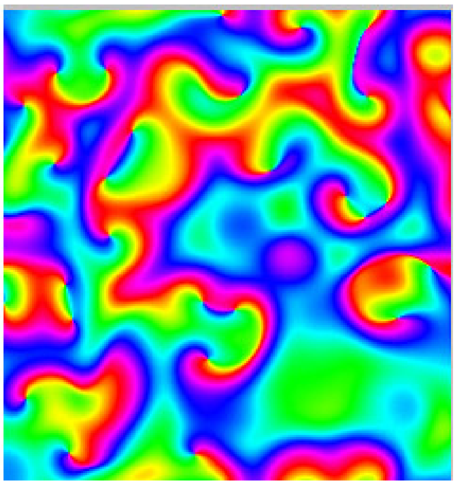


Figure 32   Configuration arising from the Swift Hohenberg equation
Figure 33   Configuration arising from application of the complex Ginsburg Landau equation

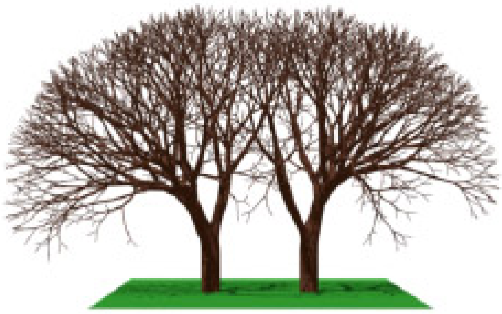


Figure 34   A sketch of mine showing a real tree as it typically is in winter, and strongly showing the positive space generated between the tree's branches
Figure 35 On the right: a context-sensitive L-system simulation of tree growth in two adjacent, interacting trees. The spaces between branches still lack the intensity as centers caused by the action of each volume of space as a system in its own right.

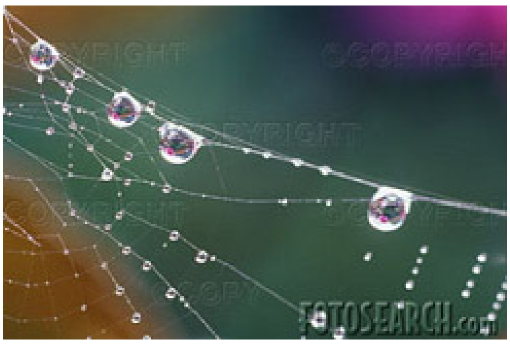


Figure 36   Dewdrops on a spiders web
Figure 37   Dewdrops on a grass stem

Figure 38   Salingaros studied 25 famous buildings by counting the number and density of local symmetries, according to the measure proposed in NOO, also adding further elaborations to estimate the density of symmetries. He found that of the buildings he studied, the Parthenon had the highest count of local symmetries, and was highest in symmetry density.

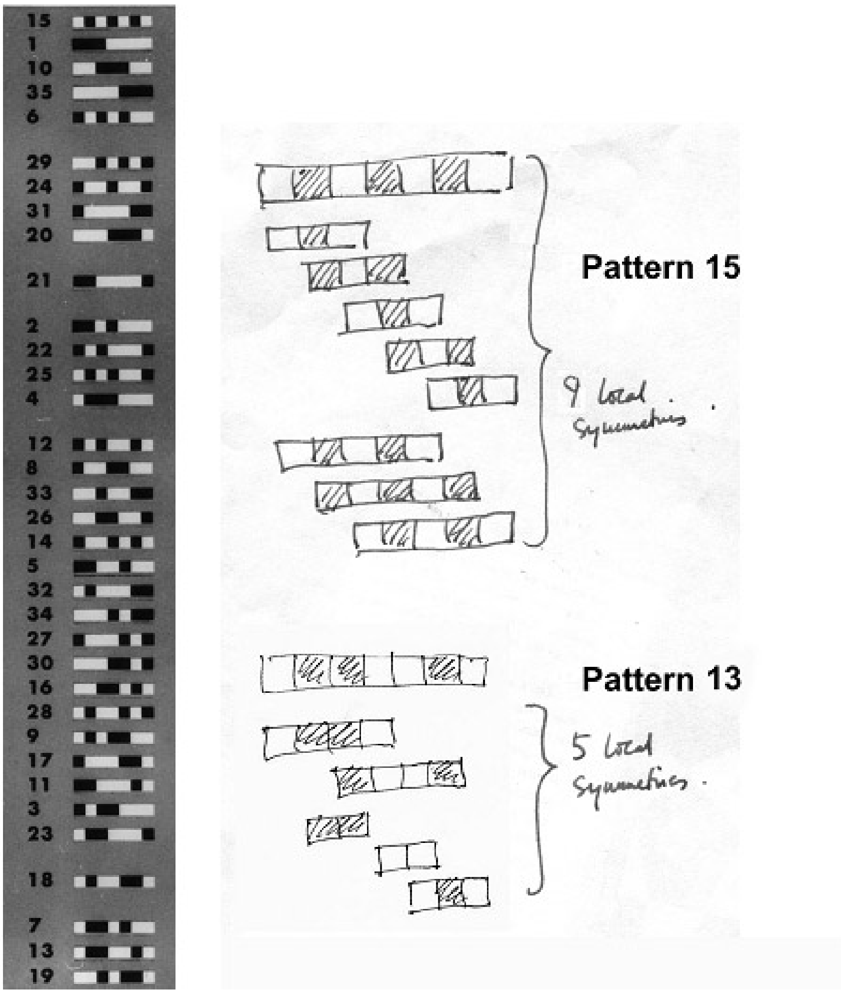
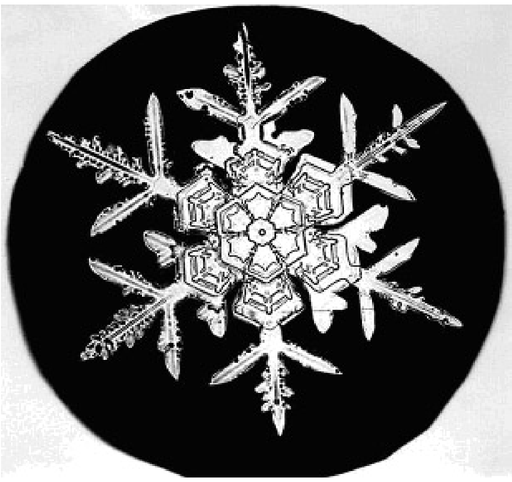


Figure 39   The local "subset" symmetries that occur in a system of 35 different strips



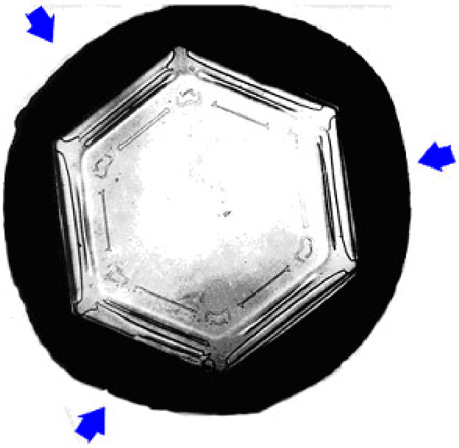


Figure 40 and Figure 41 Two snowflakes with three-fold symmetries (axial and face-oriented) that appear within the six-fold scheme.

Figure 42   A two-fold symmetry, appearing (as if by chance) within the six-fold scheme.

Figure 43   Perspective overview of a Boston project with some 200 condominums

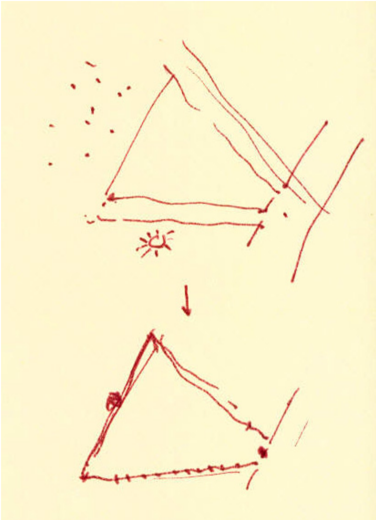


Figure 44A   Step 1

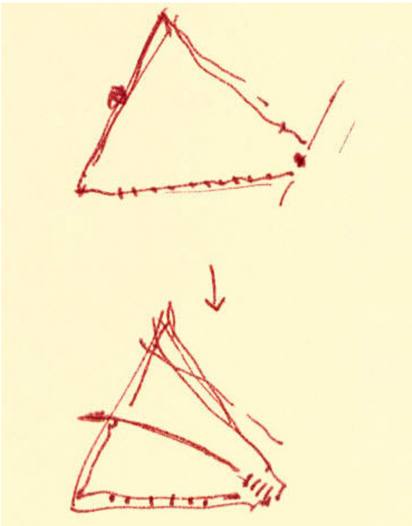


Figure 44B   Step 2

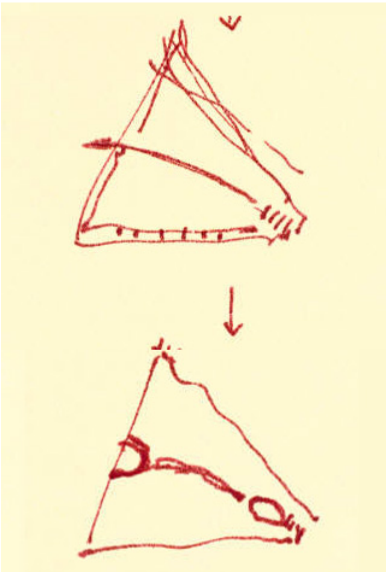


Figure 44C   Step 3

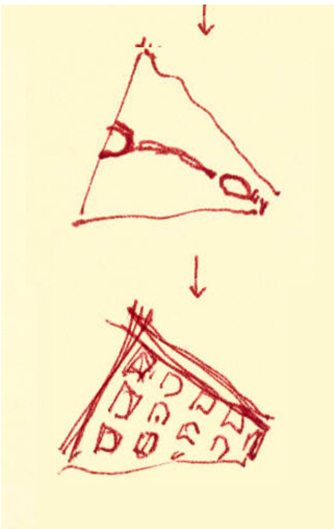


Figure 44D   Step 4

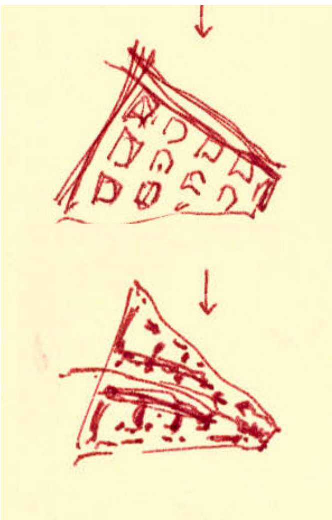


Figure 44E   Step 5

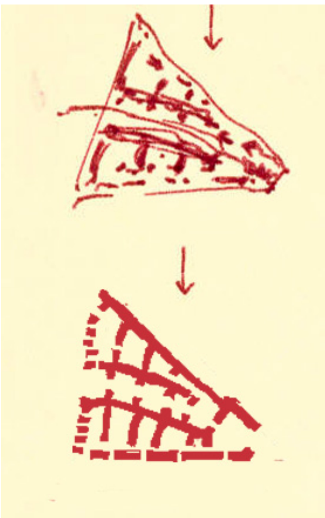


Figure 44F   Step 6

Figure 45 Final plan, after the six steps I have described, and a further twenty comparable steps (not illustrated here)

Figure 46 Different gardens in the project, each with its unique character and atmosphere

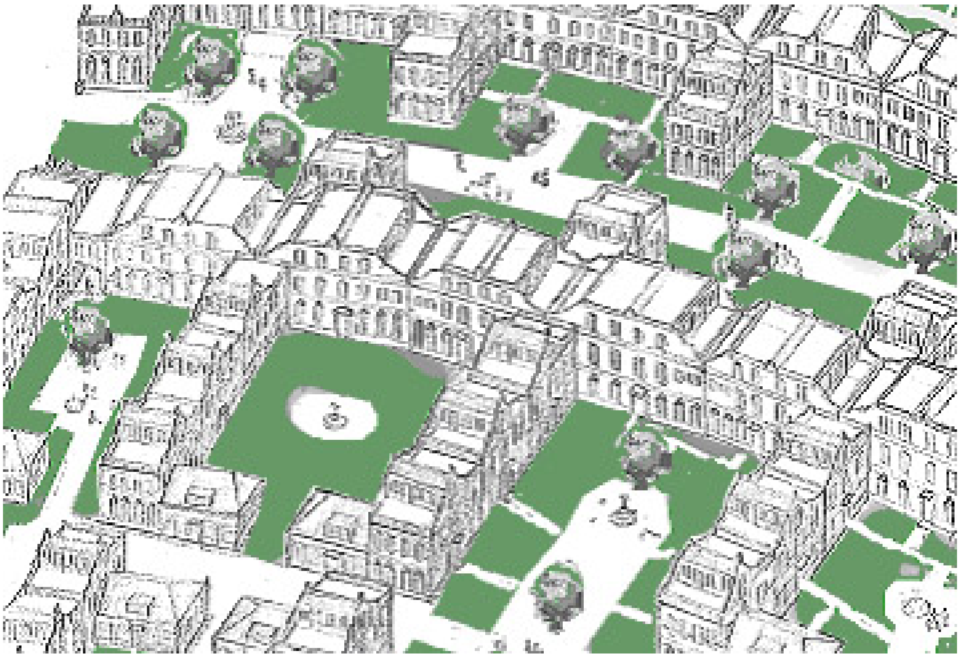


Figure 47 Detailed view of the houses and apartments, showing the detailed adaptation and variety which arises from the use of harmony-seeking computations

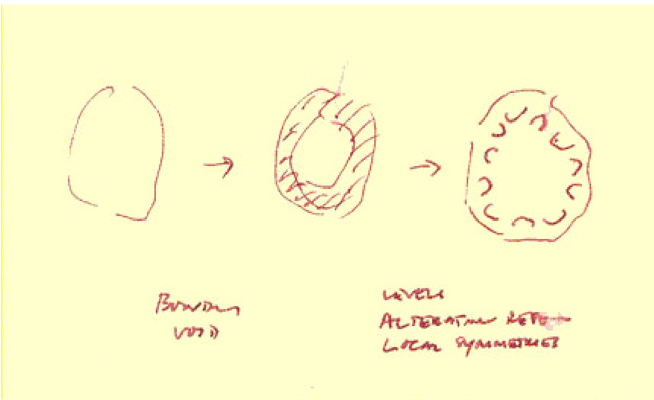


Figure 48 → THICK BOUNDARIES, THE VOID → LEVELS OF SCALE, ALTERNATING REPETITION, LOCAL SYMMETRIES

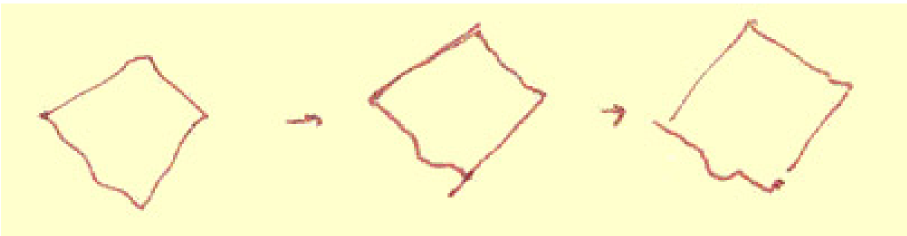


Figure 49 → STRONG CENTERS, LEVELS OF SCALE → LOCAL SYMMETRIES

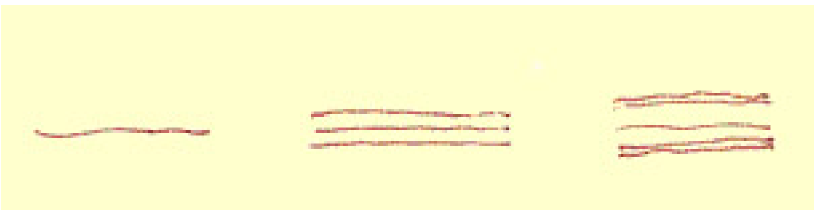


Figure 50 → THICK BOUNDARIES, THE VOID → GRADIENTS, LEVELS OF SCALE, ALTERNATING REPETITION, ECHOES

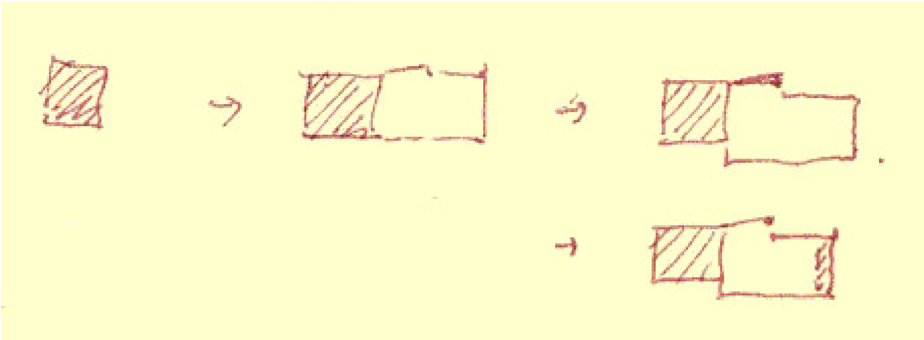


Figure 51 → POSITIVE SPACE → GRADIENTS, LEVELS OF SCALE → POSITIVE SPACE

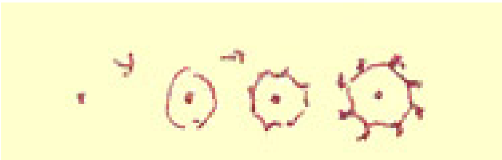


Figure 52 → STRONG CENTER → LEVELS OF SCALE → DEEP INTERLOCK, LOCAL SYMMETRIES

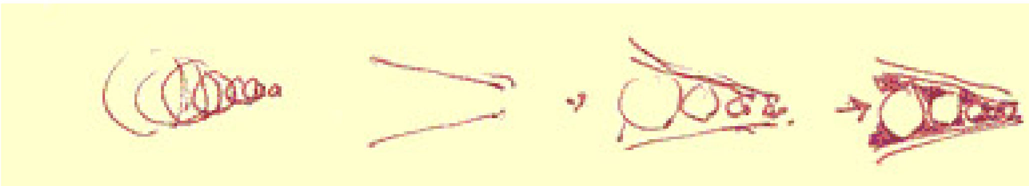


Figure 53 → GRADIENTS → SIMPLICITY → POSITIVE SPACE → LOCAL SYMMETRIES → CONTRAST